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Families of Diophantine equations

by
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Let K be a number field and $m \in K$, $m \neq 0$. For each unit $\varepsilon \in \mathbf{Z}_K^\times$, let $f_\varepsilon(X) \in \mathbf{Z}[X]$ be the irreducible polynomial of ε over $\mathbf{Q}$ and let $d = [\mathbf{Q}(\varepsilon) : \mathbf{Q}]$ be its degree. Then $F_\varepsilon(X, Y) = Y^d f_\varepsilon(X/Y) \in \mathbf{Z}[X, Y]$ is an irreducible binary form of degree d with integer coefficients. We prove that the set

$$\left\{(x, y, \varepsilon) \in \mathbf{Z}^2 \times \mathbf{Z}_K^\times \;\mid\; xy \neq 0,\ [\mathbf{Q}(\varepsilon) : \mathbf{Q}] \geq 3,\ F_\varepsilon(x, y) = m\right\}$$

is finite. In some cases the result is effective and we obtain

$$\max\{|x|, |y|, e^{\mathrm{h}(\varepsilon)}\} \leq \kappa_1 m^{\kappa_2}$$

with positive and effectively computable constants κ_1 and κ_2. Here, $\mathrm{h}(\varepsilon)$ is the absolute logarithmic height of ε.

This is a joint work with Claude Levesque.