Heights and totally real numbers

In 1973 Schinzel proved that the standard logarithmic height h on the maximal totally real field extension $\mathbb{Q}^{tr}$ of the rationals is either zero or bounded from below by a positive constant. We will give a generalization related to dynamical heights $\widehat{h}_f$ associated to rational functions f on the Riemann sphere. The main result is a complete classification of rational functions f, according to their Julia sets, such that $\widehat{h}_f$ on $\mathbb{Q}^{tr}$ is either zero or bounded from below by a positive constant.