ABSTRACT OF LECTURES IN CENTER DE GIORGI

A. VOLBERG

A theory of nonhomogeneous Calderón-Zygmund (CZ) operators is the topic of the first and the second lectures and partially also of the third one. In the first lecture we will show that the main cornerstone of the theory of CZ operators is not a cornerstone at all. Namely, one can completely get rid of homogeneity of the underlying measure. The striking application of this theory is the solution of the series of problems of Painlevé, Ahlfors and Vitushkin on the borderline of Harmonic Analysis and Geometric Measure Theory.

In the second lecture we will show how the ideas from nonhomogeneous CZ theory interplay with Tolsa's ideas of capacity theory with Calderón-Zygmund kernels to give Tolsa's solution of the famous Vitushkin conjecture of semiadditivity of analytic capacity. We also show what changes should be maid if we want to grow the dimension and to prove the semiadditivity of Lipschitz harmonic capacity in $\mathbb{R}^n, n > 2$, where the wonderful tool of Menger-Melnikov's curvature is "cruelly missing" (the expression of Guy David). We will also show why nonhomogeneous CZ theory should imply that the set of finite length and positive analytic capacity **must** have a non-trivial intersection with Lipschitz curve (we sketch the proof different from the original Guy David's proof).

In the third lecture we will try to show how the ideas of nonhomogeneous CZ theory are related to an old problem in the area of inverse spectral problems. The problem here belongs to Jean Bellissard and can be formulated as follows: let f be any hyperbolic polynomial with real Julia set $J(f)$. Let us build a Jacobi matrix by harmonic measure of $J(f)$. Then one needs to prove that this matrix is always almost periodic.

This gives a rich class of almost periodic Jacobi matrices with Cantor spectrum and singular continuous spectral measure. It has been known that for the second degree polynomials $z^2 - c$ with sufficiently large c the almost periodicity holds. For some special polynomials of degree bigger than 2, this also was known (these were some modified Tchebyshoff polynomials).

Here nonhomogeneous CZ theory comes unexpectedly in the form of two weight Hilbert transform, which appears very naturally in the problem of Bellisard mentioned above.