

A regularity result for spectral problems

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In this talk we will study the optimal sets $\Omega^* \subseteq \mathbb{R}^N$ for spectral functionals of the form $F(\lambda_1(\Omega), \dots, \lambda_k(\Omega))$, which are bi-Lipschitz with respect to each of the eigenvalues $\lambda_1(\Omega), \lambda_2(\Omega), \dots, \lambda_k(\Omega)$ of the Dirichlet Laplacian on Ω , a prototype being the problem

$$\min \{ \lambda_1(\Omega) + \dots + \lambda_k(\Omega) : \Omega \subseteq \mathbb{R}^N, |\Omega| = 1 \}.$$

We will prove the Lipschitz regularity of the eigenfunctions u_1, \dots, u_k of the Dirichlet Laplacian on the optimal set Ω^* and, as a corollary, we deduce that Ω^* is open. This will be done both employing techniques from the regularity for free boundary problems and with a two dimensional “elementary” method. For functionals depending only on a generic subset of the spectrum, as for example $\lambda_k(\Omega)$, our result proves only the existence of a Lipschitz continuous eigenfunction in correspondence to each of the eigenvalues involved.

This is a joint work with D. Bucur, A. Pratelli and B. Velichkov.