

Minimal crystallizations of 3- and 4-manifolds

Biplab Basak

23 February, 2015

Department of Mathematics, Indian Institute of Science, Bangalore 560 012, India,
biplab10@math.iisc.ernet.in.

Abstract

We have introduced the weight of a group which has a presentation with number of relations is at most the number of generators. We have shown that the number of vertices of any crystallization of a connected closed 3-manifold M is at least the weight of the fundamental group of M . We have also constructed crystallization of $L(kq - 1, q)$ with $4(q + k - 1)$ vertices for $q \geq 3, k \geq 2$ and $L(kq + 1, q)$ with $4(q + k)$ vertices for $q \geq 4, k \geq 1$. By a recent result of Swartz, our crystallization of $L(kq + 1, q)$ are minimal when $kq + 1$ are even.

Also, we have provided a minimal crystallization of the standard PL K3 surface. In combination with known results this yields minimal crystallizations of all simply connected PL 4-manifolds of “standard” type, that is, all connected sums of $\mathbb{C}\mathbb{P}^2$, $S^2 \times S^2$, and the K3 surface. In particular, we obtain minimal crystallizations of a pair of homeomorphic but non-PL-homeomorphic 4-manifolds.