Dynamics on Wild Character Varieties and Painlevé Equations - Abstract

April 13, 2015

In the first part of the lecture we will sketch a *work in progress* in collaboration with Emmanuel Paul and Julio Rebelo, based on (or related to) some results due to several people, mainly: Ph. Boalch, S. Cantat, B. Dubrovin, M. A. Inaba, K. Iwasaki, M. Jimbo, F. Loray, B. Malgrange, M. Mazzoco, T. Miwa, M. van der Put, M-H Saito, K. Ueno, E. Witten,... and the Kyoto school around T. Kawai and Y. Takei.

In the present state it is mainly a *PROGRAM* with more conjectures than proofs....

We would like to understand:

1. the *dynamics* and the *"wild dynamics"* (that is, roughly speaking, the ordinary dynamics, that is the dynamics coming from the non-linear monodromy "plus" the dynamics coming from "non linear Stokes phenomena") of equations of *isomonodromic deformations* and of *wild isomonodromic deformations* using the (*generalized*) *Riemann-Hilbert correspondances* and the corresponding (wild) dynamics on the *(wild) character varieties*;

2. the *confluence phenomena* for the equations of (wild) isomonodromic deformations and the corresponding confluence phenomena for the (wild) dynamics.

Our (long term !) aim is to built a *general theory*, testing it at each step on the case of the *Painlevé equations* (which is already far to be trivial).

Our initial motivation was to compute the *Malgrange groupoids* of the six Painlevé equations. Our conjecture is that it is *the biggest possible* (that is the groupoid of transformations conserving the area) in the generic cases (it is known for P_I: Casale, and P_{VI}: Cantat-Loray).

The classical character varieties are moduli spaces of monodromy data of regular-singular connections, that is spaces of representations of the fundamental group of a punctured (or not) Riemann surface. Boalch proved that they admit holomorphic symplectic structure.

The *wild* character varieties generalize the classical (or *tame*) character varieties. They are moduli spaces of generalized monodromy data of meromorphic connections. In the

irregular case it is necessary to add "Stokes data" to the classical monodromy. Then the wild character varieties are spaces of representations of a wild fundamental *groupoid*.

In the global irregular case it is *necessary* to use groupoids.

We will explain what is a wild character varieties associated to wild Riemann surfaces. These notions (essentially due to Boalch) are associated to the a priori data of a complex reductive linear algebraic group G. To a wild Riemann surface is associated a generalized braid group and this group induces a rational dynamic on the corresponding wild character variety (that we will call *Boalch dynamics*).

In the second part of the lecture we will describe in some details what it is happening for the case of P_{VI} and P_V ($G := Sl_2$) using systematically groupoids and Fricke coordinates. For P_{VI} it is a reformulation of results of Cantat-Loray and everything works very well. For P_V we will compute the Boalch dynamics and we will see that it is "too poor" to reflect the wild dynamics of P_V.