SCHOOL ON HYPERBOLIC DYNAMICS, 12–16 JUNE 2017 (CRM, PISA)

MINICOURSE TITLE: "YOUNG TOWERS AND SINAI-RUELLE-BOWEN MEASURES FOR NON-UNIFORMLY HYPERBOLIC SURFACE DIFFEOMORPHISMS". LECTURER: STEFANO LUZZATTO

In the 1970's, Sinai, Ruelle and Bowen introduced a revolutionary new way of studying complicated "chaotic" attractors by probabilistic and statistical methods and used finite Markov partitions to construct a special class of invariant probability measures, which we now call SRB measures, for Axiom A (uniformly hyperbolic) attractors [1]. Since then there has been a large amount of research aimed at extending their methods and results to more general classes of systems satisfying weaker forms of (non-uniform) hyperbolicity, usually however under some non-trivial additional "domination" condition between the expansion and the contraction [2, 3, 5, 4].

In this short course, depending on the amount of time available, I will review some of the history of the subject and some of the methods used and results which have been obtained. In particular I will describe a powerful generalization of Markov partitions introduced by Young [6] at the end of the 1990's and now known as Young Towers, which have been used to prove the existence of SRB measures for several specific classes of systems. I will then focus on recent joint work with Climenhaga and Pesin which shows that Young Towers exist and can be used to construct SRB measures even in the setting of surface diffeomorphisms satisfying non-uniform hyperbolicity assumptions in a very general sense, without any domination.

References

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[5] Alves, J.F.; Dias, C.L.; Luzzatto, S. Pinheiro, V. SRB measures for partially hyperbolic systems whose central direction is weakly expanding. Journal of the European Mathematical Society, To appear.

[6] Young, L.-S. (1998). Statistical Properties of Dynamical Systems with Some Hyperbolicity. The Annals of Mathematics, 147(3), 585.