

# Totally dissipative evolutions of probability measures

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We study well-posedness results for *Measure Differential Equations*, i.e. first order evolutions in the Wasserstein space  $(\mathcal{P}_2(\mathbf{H}), W_2)$  of probability measures, driven by *probability vector fields* in  $\mathcal{P}_2(\mathbf{TH})$ , with  $\mathbf{H}$  a separable Hilbert space and  $\mathbf{TH} = \mathbf{H} \times \mathbf{H}$ .

We start by presenting the classical Hilbertian theory of maximal dissipative evolutions providing well-posedness by mean of an Implicit Euler scheme. We then move to the metric setting of  $(\mathcal{P}_2(\mathbf{H}), W_2)$  and introduce a notion of pseudo-scalar product and, consequently, a metric notion of dissipativity for a probability vector field. We then discuss the obstructions, with respect to the Hilbertian case, to the implementation of an Implicit Euler scheme in this framework. To overcome these issues, we ask for a (stronger) notion of total dissipativity which allows us to lift the whole problem in a  $L^2(\Omega; \mathbf{H})$  (Hilbert) space of parametrizations.

Wasserstein gradient flows are particular examples of application of this theory.

This is a joint work with Giuseppe Savaré (Bocconi University - Italy) and Giacomo Enrico Sodini (Universität Wien - Austria).