The Structure of Turbulence in Newtonian and Viscoelastic flows: Polymers and Drag Reduction Lectures 3/4

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DIPARTIMENTO DI MECCANICA E AERONAUTICA



Introduction

▷ A well known effect of dilute polymers in wall turbulence: very large *drag reduction* up to 70% (since Toms, 1948)

- ▷ Typical applications
 - channels or pipelines (e.g. Alaska oil pipe)
 - ships and boats could have great advantages ··· but environmental problems
- Several phenomenological models have been proposed in the past to explain such phenomena (e.g Lumley 1963, De Gennes 1986, and more recently L'vov et al. 2004) · · · but a complete physical comprehension is still missing (e.g. Sreenivasan and White 2000)

Introduction

▷ Firemen water cannons with *pure water* and *dilute polymer* solution



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▷ Same power *but* different bulk velocity & throughput

Outline of the presentation

- Experimental evidence: the effect of polymers on wall turbulence
- How the polymers act: their physical model
- The numerical simulations: how reliable they are
- A scale by scale energy budget: a tool for physical understanding

The analysis and the main findings

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The analysis and the main findings

Experimental evidence: friction coefficient (I)

▷ Basic experiments by Virk (1975) show how the gross features of the flow are modified e.g. *friction coefficient*

$$f = 2 \tau_w / (\rho u_b^2)$$



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Experimental evidence: friction coefficient (II)

The decrease of drag is even more evident in the Prandtl,Karman coordinates

$$1/\sqrt{f} = U_0/u_* \qquad \sqrt{f} \ Re = \ Re_*$$



▷ In both cases it clearly appears an onset for a certain value of *Re*_{*} (or *Re*) i.e. for lower values the polymers have no effect.

Experimental evidence: mean velocity profiles

We report results by Virk (pipe flow), see also Warholic et al. (channel flow)



we see clearly the sequence of profiles for increasing concentration

Experimental evidence: mean velocity profiles

- \triangleright We observe in both sets of experimets
 - a larger throughput for the same $\tau_{\rm w}$ and increasing concentration of polymers
 - a logaritmic profile with the same slope but larger intercept: Newtonian plug

$$u_* = 2.5 \log y^+ + B$$

 a Maximum Drag Reduction (MDR) limit profile, which has a universal slope given by

$$u_* = 11.7 \log y^+ - 17$$

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strikingly insensitive to polymer species and concentration

Experimental evidence: turbulence intensity

 The axial turbulence intensity shows a substantial increase for pipe flow (Virk)



On the contrary we observe a depletion of the radial component

Experimental evidence: turbulence intensity

▷ Analogous results for channel flow (Warholic et al.)



We observe a decrease also of streamwise component for the case of larger DR

Experimental evidence: the Reynolds stresses

- ▷ In channel flow (Warholic et *al.*) we see a decrease in Reynolds stresses more and more substantial with larger DR
- ▷ The missing part to reach the total stress called *stress deficit* is given by the polymer stress τ_p

$$au > \mu rac{dU}{dy} -
ho \langle u \, v
angle$$

- Reynolds stresses become very small (approximately zero) for conditions close to MDR
- Analogous results in pipe flow (Virk)



Experimental evidence: the onset

- ▷ The onset of drag reduction occurs at a rather well defined critical value of the wall shear stress τ_w^c
- \triangleright The *onset* is correlated with the random coil size $R_0 \ll \eta$

$$\tau_{w}^{c} = u_{*,c}^{2}\rho = CR_{0}^{-3}$$
the critical viscous time
$$t_{*}^{c} = \frac{\nu}{u_{*,c}^{2}} = \frac{\mu}{\tau_{w}^{c}} = \frac{\mu}{C}R_{0}^{3}$$
the relaxation time
(Zimm)
$$t_{p} = \alpha R_{0}^{3}$$
Time criterion (Lumley)
$$t_{p} \simeq t_{*}^{c}$$

$$t_{p}^{c} \simeq t_{*}^{c}$$

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Experimental evidence: the onset

▷ *De*_{*} is the relevant parameter for the *onset*

$$De^c_* = rac{t_{
ho}}{t^c_*} = rac{lpha R_0^3}{\mu/cR_0^3} = const.$$



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Physical aspects of drag reducing polymers

▷ Polymers are long linear chains of *monomers*

$$R_c = n b$$
$$(\simeq 6000 \quad nm)$$



> At equilibrium they are random coils with

$$R_o \propto \left(nb^2
ight)^{1/2}$$

gyration radius $(\simeq 100 \ nm)$



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 The coil deforms under shear and recovers equilibrium conditions by thermal motion (entropic elasticity)

Physical aspects of drag reducing polymers

- \triangleright The relaxation to equilibrium is ruled by a spectrum characteristic of times $t_1 > t_2 > \cdots$
- \triangleright Only the principal relaxation time $t_p = t_1$ is relevant for turbulence
- ▷ This opens the way for an ultra-simplified description of the polymer, as a system with a single internal degree of freedom: the elastic dumbbell



The micro-mechanics of a dumbbell

- \triangleright Forces acting on the dumbbell
 - Idrodynamic forces on the beads

$$\mathbf{F}_A = f(\mathbf{u}_A - \dot{\mathbf{x}}_A) \\ \mathbf{F}_B = f(\mathbf{u}_B - \dot{\mathbf{x}}_B)$$

Elastic force

$$\mathbf{F}_{AB} = k(\mathbf{x}_A - \mathbf{x}_B)$$

▷ The mass of the beads is negligible

$$(\mathbf{u}_A - \mathbf{u}_B) = \nabla \mathbf{u}|_G(\mathbf{x}_A - \mathbf{x}_B) \quad \Rightarrow \quad \dot{\mathbf{R}} = \mathbf{R} \cdot \nabla \mathbf{u} - \frac{1}{t_p} \mathbf{R}$$

with $t_{\rho} = f/k$ relaxation time

To account for the thermal motion a Brownian forcing is added

$$\dot{\mathbf{R}} = \mathbf{R} \cdot \nabla \mathbf{u} - \frac{1}{t_{\rho}} \left(\mathbf{R} - \boldsymbol{\xi} \right)$$



The polymer stresses

- \triangleright Given the dumbbel ensemble with *number density* n_p
- the force through a surface

$$\mathbf{t} = n_{p} \langle \mathbf{F} \, \mathbf{R}
angle \cdot \mathbf{n}$$

hence

$$\mathbf{F} = -\frac{3k_B\theta}{nb^2}\mathbf{R}$$



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The polymer stresses

▷ In terms of the conformation tensor $\mathcal{R} = \langle \mathbf{R} \otimes \mathbf{R} \rangle$, the extra-stress is

$$\mathbf{T}_{p} = \frac{n_{p}k_{B}\theta}{R_{0}^{2}/3} \langle \boldsymbol{\mathcal{R}} \rangle$$

where $nb^2 = \langle R^2 \rangle_0 = R_0^2$

Since at equilibrium condition the extra-stress vanishes

$$\mathbf{T}_{\rho} = \frac{\nu_{\rho}}{t_{\rho}} \left(\frac{\langle \boldsymbol{\mathcal{R}} \rangle}{R_0^2/3} - \mathbf{I} \right)$$

where t_p is the relaxation time and ν_p is the contribution to viscosity due to polymers (Oldroyd-B)

▷ For finite extension non linear elasticity (FENE-P)

$$\mathbf{T}_{p} = \frac{\nu_{p}}{t_{p}} \left(f \frac{\langle \boldsymbol{\mathcal{R}} \rangle}{R_{0}^{3}/3} - \mathbf{I} \right)$$

and $f = f(\langle R^2 \rangle)$ in the Peterlin approximation

The equation for the conformation tensor

 $\triangleright\,$ From the force balance, by averaging, we obtain the evolution equation for ${\cal R}\,$ in dimensioless form

$$\frac{D\boldsymbol{\mathcal{R}}}{Dt} = \boldsymbol{\mathcal{R}} \cdot \nabla \mathbf{u} - \nabla \mathbf{u}^{T} \cdot \boldsymbol{\mathcal{R}} - \frac{1}{De^{*}} \left(\boldsymbol{\mathcal{R}} - \mathbf{I} \right)$$

▷ The model is completed by momentum equation for an incompressible flow ($\nabla \cdot \mathbf{u} = 0$), where the extra-stress \mathbf{T}_p is included

$$\frac{D\mathbf{u}}{Dt} = -\nabla \rho + \frac{1}{Re^*} \nabla^2 \mathbf{u} + \nabla \cdot \mathbf{T}_{\rho}$$

with

$$De^* = rac{ au_p}{u_*^2/
u}$$
 $Re^* = rac{hu_*}{
u}$ $\mathbf{T}_p = rac{
u_p}{De^*} (\mathcal{R} - \mathbf{I})$

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Energy balance for the polymers

 \triangleright If we take the trace of the evolution equation for \mathcal{R} we obtain for the elastic energy of the dumbbell population

$$\frac{DE_p}{Dt} = \Pi_p - \epsilon_p$$

where

$$E_{p} = \frac{\nu_{p}}{Re^{*}} \frac{1}{2} Tr(\mathcal{R}) \quad \Pi_{p} = Tr\left(\mathcal{R} \cdot \nabla \mathbf{u}\right) \frac{\nu_{p}}{Re^{*}}$$
$$\epsilon_{p} = \frac{1}{De^{*}} \frac{1}{2} Tr(\mathcal{R}) \frac{\nu_{p}}{Re^{*}} = \frac{E_{p}}{Re^{*}}$$

with

- Π_p energy transfer to polymer microstructure
- ϵ_p total (average+fluctuation) dissipation by the polymers

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Energy balance for the solvent

The evolution equation for the total (average + fluctuation) kinetic energy of the carrier fluid is

$$\frac{DE_k}{Dt} = -\nabla \cdot (p\mathbf{u}) + \frac{1}{Re^*} \nabla \cdot \left[\left(\nabla \mathbf{u} + \nabla \mathbf{u}^T \right) \cdot \mathbf{u} \right] - \epsilon_N + \nabla \cdot (\mathbf{T}_p \cdot \mathbf{u}) - \Pi_p$$

again Π_p is the energy transferd to the polymers

 \triangleright By combining the two eqs, we obtain the equation for $E_T = E_k + E_p$

$$\frac{DE_T}{Dt} = \nabla \cdot \Phi - \epsilon_T$$

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where $\epsilon_T = \epsilon_N + \epsilon_p$

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Numerical simulation

- The physical model reproduces the gross features seen in the experiments
 - mean velocity profile
 - Reynolds stresses
 - turbulence intensities
 - various components of the energy balance
 - correlations and the shape of the structures
- ▷ The increasing effect of polymers is here explored by increasing values of $De_* = t_p/t_*$

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Mean profile and turbulent fluctuations

- \triangleright A series of Newtoniamn plugs at increasing values of De_*
- Fluctuations
 - streamwise increase
 - normal to wall decrease



 \triangleright Channel flow at $Re_* = 300$

Reynolds stresses & Drag Reduction

 \triangleright Reynolds stresses decrease with De_*

 \triangleright DR increases, almost linearly, with De_* up to MDR



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The energy balance (I)

The T.K.E. shows the spatial redistibution due to inhomogeneity and the turbulence-polymers interaction

$$-\frac{d\Phi}{dy} - \frac{d}{dy} \langle T'_{p} \cdot u' \rangle - \langle u' v' \rangle \frac{dU}{dy} - \langle \epsilon_{N} \rangle - \langle \pi_{p} \rangle = 0$$



The energy balance (II)

The T.K.E. shows the spatial redistibution due to inhomogeneity and the turbulence-polymers interaction

$$-\frac{d\Phi}{dy} - \frac{d}{dy} \langle T'_{p} \cdot u' \rangle - \langle u' v' \rangle \frac{dU}{dy} - \langle \epsilon_{N} \rangle - \langle \pi_{p} \rangle = 0$$



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The striking effect on large scales

- DNS gives a good tool to analyze the alteration of turbulence due to polymers
- Polymers drain energy at small scales from the inertial cascade (see results fro isotropic turbulence De Angelis et al. 2005)
- However drag reduction is due to their ability to modify the large scales
- Sophisticated data analysis (POD) gives an expansion in terms of empirical, most energetic, modes

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The striking effect on large scales

▷ The shape of the modes remain quite the same (De Angelis et al. 2003) however the amplitude change substantially with De



The largest modes (in terms of scale) become more energetic w.r.t. Newtonian flow. *How it occurs?*

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Scale by scale budget: physical picture



Leads to a combined analysis of energy flux both in physical space (as the T.K.E. in wall turbulence) and in the space of scales (as Kolmogorov eq. for H.I. turbulence)

Scale energy

To this purpose we consider the second order structure function

$$\delta \mathbf{u} = \mathbf{u}'(\mathbf{x} + \mathbf{r}) - \mathbf{u}'(\mathbf{x})$$

$$\triangleright$$
 energy at scale r

$$\langle \delta u^2(r|Y_c) \rangle = \langle \delta u_i \delta u_i \rangle$$

> at large scale velocities are uncorrelated

$$\lim_{r\to\infty} \langle \delta u^2(r|Y_c) \rangle = 2 \langle u'_i u'_i \rangle$$



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 \triangleright We define $f^* = \left[f'(\mathbf{x} + \mathbf{r}) + f'(\mathbf{x})\right]/2$

Kolmogorov equation

- Starting from the Navier-Sokes eqs. it may be obtained a generalized form of the Kolmogorov equation (See Hill 2001, Yakhot 2001, Danaila et al. 2001, Marati et al. 2004) which is here extended to polymer solutions
- ▷ In homogeneous conditions it reduces to

$$\nabla_{\mathbf{r}} \cdot \langle \delta u^2 \delta \mathbf{u} \rangle = -4 \langle \epsilon \rangle + 2\nu \nabla_{\mathbf{r}}^2 \langle \delta u^2 \rangle$$

and in terms of longitudinal stucture functions $\delta u_{\parallel} = \delta \mathbf{u} \cdot \mathbf{r}/r$ give the well known *Four-fifth* law

$$\langle \delta u_{\parallel}^{3}
angle = -\frac{4}{5} \langle \epsilon
angle r$$

which describes the energy cascade across scales in isotropic conditions

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A generalized form of the Kolmogorov equation

> Extended to polymers and in conservative form

$$abla_r \cdot \Phi_r(r, Y_c) + rac{d\Phi_s}{dY_c} = s(r, Y_c)$$

whith the flux Φ_r in the space of scales

$$\Phi_{r} = \frac{1}{4} \left(\langle \delta u^{2} \delta \mathbf{u} \rangle - 2\nu \nabla_{r} \langle \delta u^{2} \rangle + \langle \mathbf{T}_{p}^{*} \cdot \delta \mathbf{u} \rangle \right),$$

the flux Φ_s in physical space

$$\Phi_{s} = \frac{1}{4} \left(\langle \delta u^{2} v^{*} \rangle + \frac{2}{\rho} \langle \delta \rho \delta v \rangle - \frac{\nu}{2} \frac{d \langle \delta u^{2} \rangle}{dY_{c}} + \hat{y} \cdot \langle \delta \mathbf{T}_{\rho} \cdot \delta \mathbf{u} \rangle \right)$$

and the source term

$$s(r, Y_c) = -\frac{1}{2} \langle \delta u \delta v \rangle \left(\frac{dU}{dY} \right)^* - \langle \epsilon_N^* \rangle - \langle \pi_p^* \rangle$$

▷ For $r \to \infty$ we recover the single point TKE balance

The physical picture with polymers



▷ The scale energy is partly drained by polymers from the energy cascade

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▷ We use for the discussion a coincise form of the *r* − averaged Kolmogorov equation

$$T_r(r, Y_c) + P_e(r, Y_c) = E_e(r, Y_c) + G_e(r, Y_c) + E_p(r, Y_c)$$

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 \triangleright We use for the discussion a coincise form of the r- averaged Kolmogorov equation

$$T_r(r, Y_c) + P_e(r, Y_c) = E_e(r, Y_c) + G_e(r, Y_c) + E_p(r, Y_c)$$

where the inertial transfer is

$$T_r(r, Y_c) = \frac{1}{4} \nabla_r \cdot \langle \delta u^2 \, \delta \mathbf{u} \rangle$$

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 \triangleright We use for the discussion a coincise form of the r- averaged Kolmogorov equation

$$T_r(r, Y_c) + P_e(r, Y_c) = E_e(r, Y_c) + G_e(r, Y_c) + E_p(r, Y_c)$$

the effective production is

$$P_e(r, Y_c) = \frac{1}{2} \left[\langle \delta u \, \delta v \rangle \left(\frac{dU}{dy} \right)^* + \frac{1}{2} \frac{d \langle \delta u^2 \, v^* \rangle}{dy} + \frac{1}{\rho} \frac{d \langle \delta p \, \delta v \rangle}{dy} \right]$$

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 \triangleright We use for the discussion a coincise form of the r- averaged Kolmogorov equation

$$T_r(r, Y_c) + P_e(r, Y_c) = E_e(r, Y_c) + G_e(r, Y_c) + E_p(r, Y_c)$$

the scale energy flux due to polymers

$$G_{e}(r, Y_{c}) = \left[\nabla_{r} \cdot \langle \mathbf{T}_{p}^{*} \cdot \delta \mathbf{u} \rangle + \frac{1}{4} \frac{d}{dy} \left(\hat{y} \cdot \langle \delta \mathbf{T}_{p} \cdot \delta \mathbf{u} \rangle \right) \right]$$

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 \triangleright We use for the discussion a coincise form of the r- averaged Kolmogorov equation

$$T_r(r, Y_c) + P_e(r, Y_c) = E_e(r, Y_c) + G_e(r, Y_c) + E_p(r, Y_c)$$

the effective dissipation due to viscosity

$$E_e(r, Y_c) = \frac{\nu}{2} \left(\nabla_r^2 \langle \delta u^2 \rangle + \frac{1}{4} \frac{d^2}{dY_c^2} \langle \delta u^2 \rangle \right) - \langle \epsilon_N^* \rangle$$

▷ We use for the discussion a coincise form of the *r*− averaged Kolmogorov equation

$$T_r(r, Y_c) + P_e(r, Y_c) = E_e(r, Y_c) + G_e(r, Y_c) + E_p(r, Y_c)$$

the energy drained by polymers and eventually dissipated

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$$E_p(r, Y_c) = -\langle \pi_p^* \rangle$$

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Polymers & the alteration of turbulence: scale energy

 \triangleright Scale energy in the plane x - z as function of wall normal distance and separation



▷ The maximum of scale energy moves towards large scales, i.e. the coherent structures grow with De_{*} up to MDR

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Polymers & the alteration of turbulence: scale energy

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Polymers & the alteration of turbulence: scale energy

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 \triangleright Effective production in the plane x - z



The maximum of production decreases, becomes thicker in y and moves towards larger scales

 \triangleright Effective production in the plane x - z



The maximum of production decreases, becomes thicker in y and moves towards larger scales

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 \triangleright Effective production in the plane x - z



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The maximum of production decreases, becomes thicker in y and moves towards larger scales

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 \triangleright Effective production in the plane x - z



The maximum of production decreases, becomes thicker in y and moves towards larger scales

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 \triangleright Energy drained by the polymers in the plane x - z



 \triangleright Energy drained by the polymers in the plane x - z



 \triangleright Energy drained by the polymers in the plane x - z



 \triangleright Energy drained by the polymers in the plane x - z



Scale budget for Newtonian flows



- Production
- Inertial transfer
- - Viscous diffusion & dissipation

(Marati et al. 2004)

-

Production vs inertial transfer



Production (solid) and Inertial transfer (dash-dotted) ZPG Boundary layer $Re_{\tau} = 1100, y^+ = 100$ $\mathbf{r} = (r_x, 0, r_z) r = \sqrt{r_x^2 + r_y^2}$

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Experiments, dual plane stereo PIV



Production (solid) and Inertial transfer (dash-dotted) ZPG Boundary layer $Re_{\tau} = 1100, y^+ = 100$ $\mathbf{r} = (r_x, 0, r_z) r = \sqrt{r_x^2 + r_y^2}$ N.Saikrishnan, E.K. Longmire, I. Marusic

 \triangleright Inertial transfer in the plane x - z





 \triangleright Inertial transfer in the plane x - z

▷ Inertial transfer in the plane x - z

300 200 ***** 100 100 200 **r** 300 400

 \triangleright Inertial transfer in the plane x - z

300 200 ***** 100 100 200 **r** 300 400

 \triangleright Inertial transfer in the plane x - z

Final Remarks



Log layer

- Effective production
- Inertial transfer
- Polymer transfer
- Polymer dissipation
- - Effective Newtonian dissipation



Buffer layer

- In the log-layer polymers interact mainly with inertial transfer (as in H.I. turbulence)
- In the buffer layer polymers interact directly with the production

Final Remarks



The inverse cascade region grows in size and the maximum moves away from the wall, i.e. the buffer region, here more properly elastic layer, enlarges

Bubbly turbulent flows

Flat Plate

> L=7 m, W =0.8 m, H= 0.06 m
>
$$U_{\infty} = 0.5 \div 2 \text{ m/s}$$





The effect of bubbles



Mean velocity profile in the ZPG boundary layer

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