

Research Trimester on Dynamical Systems.

February 4 - April 26, 2002

The primary goal of this conference was to provide a state of the art description of the subject accessible to young mathematicians wishing to do research in this area as well as mathematicians and scientists from other disciplines interested in learning some of the most recent developments in the field. Moreover during the trimester the current state of the subject and some of its most recent applications, as well as major open problems, were presented to specialists in dynamical systems (and to mathematicians or scientists from other disciplines, so as to help chart a landscape for research in the area over the next years).

The following broad areas were covered: non-uniformly and partially hyperbolic systems, quasiperiodic orbits, holomorphic dynamics and foliations, interaction between dynamical systems and physics (including celestial mechanics).

Four types of lecture series catering to the different possible audiences were given:

- in-depth courses (8-10 hours)
- minicourses (about 5 hours)
- survey lectures (2-3 hours)
- research talks (1 hour)

BRIEF DESCRIPTION OF THE COURSES:

Courses, minicourses and survey lectures were offered by leading experts.

Here is a short description of the main contributions.

Holomorphic Dynamics and Foliations:

John Milnor (SUNY, Stony Brook) gave an introduction to the technique of mating Julia sets. He discussed the case of dendrites in detail: these matings give rise to new examples of rational functions whose Julia set is the whole Riemann sphere. *Charles Favre* (CNRS, Paris VII) in an 8 hours long lecture series discussed the dynamics of polynomial maps of the complex n -dimensional projective space. He showed how to generalize Brolin's theorem on the existence of an invariant probability measure limit of the distributions of the preimages of a generic point to this multidimensional case. *Marco Brunella* (CNRS, Dijon) during an 8 hours long lecture series outlined the classification of foliations on complex algebraic surfaces, with a special emphasis on non-generic cases. *Giovanni Forni* (Northwestern University) gave 5 lectures on the ergodic properties of measured foliations on Riemann surfaces and outlined the proof of genericity (in the measure category) of unique ergodicity via Teichmüller theory.

Hamiltonian Dynamics:

John Mather, in a 4 hours long lecture series gave a sketch of the proof of genericity of Arnold diffusion, thus announcing the solution of a longstanding conjecture in the field. *Antonio Giorgilli* (University of Milano Bicocca) and *Luigi Chierchia* (University of Roma III) gave two courses of 10 hours each on Nekhoroshev theory and KAM theory respectively. In these lectures, without assuming any previous knowledge of Hamiltonian perturbation theory, the stability problem of quasi integrable Hamiltonian systems was thoroughly

discussed both in finite and infinite time scales. *David Sauzin* (CNRS, Paris), in a series of 3 lectures, showed how to extend the theory of Nekhoroshev to the case of Hamiltonians with only Gevrey regularity and discussed the optimality of the results obtained.

Small divisor problems:

Bassam Fayad (CNRS, Paris XIII), in a 5 hours lecture series, discussed the relation between small divisors and ergodic theory. This relationship was first pointed out by Kolmogorov 50 years ago in the context of the study of the conjugacy classes of time-reparametrizations of linear flows on tori. Time-reparametrizations give rise to linear cohomological equations: the parameter-dependence in the one-dimensional analytic case has been treated by *David Sauzin* (CNRS, Paris) in a two hour lecture. *Raphael Krikorian* (École Polytechnique), in a 5 hours minicourse, illustrated several local and global small divisor problems arising from the theory of reducibility and Lyapunov exponents. *Stefano Marmi* (Udine and SNS), during an 8 hours lecture series, discussed the optimal arithmetical conditions for the problem of linearization of germs of one-dimensional holomorphic diffeomorphisms and studied the related quantitative problems on the size of Herman rings and Siegel disks. These problems have also been discussed in the review lecture of *Pierre Moussa* (CEA Saclay). In a 5 hours minicourse *Laurent Stolovitch* (Toulouse) showed how to generalize the notion of completely integrable system to the context of normal forms for (commuting systems of) holomorphic vector fields or germs.

Celestial Mechanics:

In a 5 hours minicourse *Alain Chenciner* (Paris VII) illustrated several new periodic solutions to the n -body problem obtained by means of a careful study of its symmetries and the use of appropriate constrained variational principles. Singularities of the n -body problem which do not arise from collisions have been the object of the 8 hours course of *Joe Gerver* (Rutgers University). *Jacques Laskar* (CNRS, Paris) gave a 4 hours long introduction to the numerical study of planetary systems, also illustrating several implications to climatology and geology.

Ergodic theory and randomly perturbed systems:

Jerome Buzzi (CNRS, École Polytechnique) gave a series of 3 survey lectures on the problem of local and global complexity of dynamical systems. *Dima Dolgopyat* (Pennsylvania State University) gave a 4 hours long introduction to the study of the dynamics of random diffeomorphisms products, whereas the 5 lectures of *Yuri Kifer* (Hebrew University) were devoted to the more classical topic of random perturbations of deterministic systems. Invariant measures and the problem of convergence to equilibrium have been the object of a 5 hours minicourse held by *Carlangelo Liverani* (Roma II).

Periodic Orbits and zeta functions:

Counting periodic orbits has been one of the first problems addressed in dynamics: the 5 hours minicourse by *Vadim Kaloshin* (MIT) served as a complete introduction to the most important recent developments in the subject, providing a sketch of the proof of topological genericity but measure-theoretical exceptionality of the superexponential growth. The two survey lectures of *Viviane Baladi* (IHES) also addressed the relationship between kneading operators and Ruelle-Lefschetz zeta functions. Topology and dynamics: *Patrice Le Calvez* (Paris XIII) gave an 8 hours long course on the dynamics of diffeomorphisms of the torus and the annulus. Special emphasis was put on the construction and properties of generalized Aubry-Mather sets to this not necessarily twist and/or area-preserving context.

Partially Hyperbolic and Non-Uniformly Hyperbolic systems:

Marcelo Viana and *Jairo Bochi* (IMPA) in a 8 hours lecture series addressed the problem of abundance of positive Lyapunov exponents. After an introduction and a complete proof of a special case of Oseledec's theorem several very recent results on this major open problem were proved. Rates of mixing in non-uniformly hyperbolic systems have been discussed by *Stefano Luzzatto* (Imperial College) in a short minicourse (3 hours).

Bifurcation theory and related topics:

The central role of the unfolding of homoclinic bifurcations in the understanding of generic properties of dynamics has been the object of the survey by *Jacob Palis* (IMPA). *Jean-Christophe Yoccoz* (Collège de France) gave a short minicourse on this subject, discussing the proof of the generalization of the classical Palis-Takens theorem to the case of "large" Hausdorff dimension of the intersection of the invariant Cantor sets. The 6 hours long course by *Carlos Gustavo Moreira* (IMPA) centered on the properties of dynamics which depend on the dimension of the intersections of invariant Cantor sets. An important application is to the study of Lagrange and Markov spectrum of an irrational number. An important class of examples with non-trivial dynamics after (a cascade of) bifurcations is provided by unimodal maps, which have been the object of the 3 survey lectures by *Arthur Avila De Melo* (Collège de France).

Dynamics on non-archimedean fields:

A new important branch of holomorphic dynamics is the study of iteration of rational maps with coefficients in a non-archimedean field: *Jean-Christophe Yoccoz* (Collège de France) gave two introductory survey lectures on this topic. A more systematic approach was taken by *Juan Rivera* (SUNY, Stony Brook) in his 6 hours minicourse. Not only the major open problems have been presented but also the relationship with number theory, rigid analytic geometry and the arithmetic of elliptic curves.

SEMINARS:

About 20 seminars from invited participants have been given.

PARTICIPANTS:

About 80 junior participants from 16 countries have attended the various activities of the trimester. Among them 27 have received a grant (partially) covering travel and/or living expenses. The nationalities of the participants are: Russia, Israel, Iran, France, Mexico, Brasil, Slovakia, Britain, Portugal, Romania, Japan, Tchech, Poland, China, U.S., Italy.

FINANCIAL SUPPORT:

The cost of all the invited speakers has been covered by a grant from the Istituto Nazionale di Alta Matematica, in the framework of the program "Bimestri e trimestri intensivi di ricerca". Most invited talks and some of the reimbursements to junior participants have been also financed by INDAM. Financial support by the European Science Foundation PRODYN program has been used to support the travel and/or living expenses of 18 participants.

