

INTENSIVE RESEARCH TRIMESTER  
"GEOMETRIC FLOWS AND GEOMETRIC OPERATORS"  
PISA, SPRING 2009

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The trimester will be hosted and organized by the Centro di Ricerca Matematica "Ennio De Giorgi" in Pisa, Scuola Normale Superiore, during May, June and July 2009, in collaboration with INdAM, GNFm, Institut Joseph Fourier, Grenoble (FR) and Project ANR "Flot et Operateurs Geometriques" ANR-07-BLAN-0251-01, SISSA – Trieste, FIRB Ideas "Analysis and Beyond".

Website: <http://cvgmt.sns.it/GFO>

### 1. PRESENTATION OF THE TRIMESTER

The trimester will concentrate on two main topics, related to series of lectures at introductory level.

The first one is the study of geometrically invariant operators, possibly of higher order. In recent years, there has been an intensive study of the relations between this kind of operators (which satisfy some invariance property under conformal change of metric), their associated scalar invariants and the study of the related partial differential equations. In two dimensions for example, the integral of the Gauss curvature (whose transformation law under conformal changes of metrics is related to the Laplace operator) determines completely the topology of the surface. In four dimensions there are partial analogues, involving the  $Q$ -curvature and the Paneitz operator, but most results are limited to the case of manifolds with positive curvature: it would be interesting to relax these assumptions and consider more general situations.

Lectures related to this research direction will be given by Matt Gursky, Jeff Viaclovsky and Andrea Malchiodi.

The second topic will concern geometric flows, in particular, higher order flows, "coupling" of flows and relations to theoretical physics.

Lei Ni will present recent progresses in applying Ricci flow to the study of the structure of Riemannian and Kähler manifolds.

Mauro Carfora will discuss some recent ideas in setting connections between geometric flows and theoretical physics, in particular with quantum field theories, string theory and renormalization group techniques.

A workshop putting together leading theoretical physicists and mathematicians is planned for the second part of June.

In July we plan to organize a workgroup on mean curvature flow with the aim to follow the "classical" line of analysis of the evolution till the first singular time (the works of Ecker,

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Huisken, Hamilton, etc).

The recent results about Ricci flow gave a strong impulse to the field of evolution of geometric structures, but not so much is known at the moment about other “natural” flows which can be used analogously to find out canonical metrics or to investigate the geometry and topology of manifolds, in particular in higher dimensions.

One of the goal of the trimester is to try to generalize these techniques, considering other flows associated to geometric quantities like functionals of the curvatures, with special care for the ones which are conformally invariant, for instance, the  $Q$ -curvature.

This analysis leads to higher order flows and it is one of our motivations to put together people working on higher order differential operators and on geometric flows.

At the end of June there will be the general conference of the trimester.

We plan, in parallel to the series of lectures, to organize daily seminars given by the guests at the Center De Giorgi. All the people will be invited to give a talk on their research activities.

Financial support is available for young students. Interested people need to apply at the website of the trimester.

The official website of the trimester is <http://cvgmt.sns.it/GFO>, where you can find information about registration and application forms.

There is a mailing list for participants or interested people to get up-to-date information on the trimester at <http://cvgmt.sns.it/gfopisa>

If you have problems in registering to the list, please write an email to [c.mantegazza@sns.it](mailto:c.mantegazza@sns.it), possibly putting GFO in the subject.

There will be some possibly related events for interested people, during the semester

- *Indam School on symmetry for elliptic PDEs, Roma Univ “La Sapienza”, May 25–29, 2009*  
<http://www.mat.uniroma2.it/valdinoc/scuola-indam.html>  
*Lectures by Xavier Cabré, Manuel del Pino, Ermanno Lanconelli and Ovidiu Savin*
- *Fifth Summer School in Analysis and Applied Mathematics*  
*Rome – Univ. “La Sapienza”, June 1–5, 2009*  
<http://www.mat.uniroma1.it/people/garroni/School2009.html>  
*Lectures by Giovanni Alberti, Stefan Hildebrandt, László Székelyhidi Jr.*

## 2. SCIENTIFIC COMMITTEE

Luigi Ambrosio – SNS, Pisa

Gérard Besson – Institut Joseph Fourier, Grenoble

Jean-Pierre Bourguignon – IHES, Bures-sur-Yvette

Sun-Yung Alice Chang – Princeton

Gerhard Huisken – Max-Planck-Institut für Gravitationsphysik, Albert Einstein Institute, Golm

### 3. ORGANIZING COMMITTEE

Luigi Ambrosio – SNS, Pisa  
 Zindine Djadli – Institut Joseph Fourier, Grenoble  
 Carlo Mantegazza – SNS, Pisa  
 Andrea Malchiodi – SISSA, Trieste

### 4. MAIN CALENDAR

Week	Events
<b>May</b>	
Monday 4 – Friday 8	Gursky Lectures I
Monday 11 – Friday 15	Gursky Lectures II
Monday 18 – Friday 22	Carfora Lectures I
Monday 25 – Friday 29	Carfora Lectures II
<b>June</b>	
Monday 1 – Friday 5	—
Monday 8 – Friday 12	Malchiodi/Viaclovsky Lectures
Monday 15 – Friday 19	Ni Lectures
Monday 22 – Friday 26	Workshop on GeoFlows in Math/Physics
Monday 29 – Friday 3	General Conference “GFO in Pisa”
<b>July</b>	
Monday 6 – Friday 10	WorkGroup on Mean Curvature Flow I
Monday 13 – Friday 17	WorkGroup on Mean Curvature Flow II
Monday 20 – Friday 24	—

### 5. ABSTRACTS OF THE LECTURES

**Matt Gursky**, University of Notre Dame – *Fully Nonlinear Equations in Conformal Geometry*

Our goal in this series of lectures is to provide a comprehensive introduction to the  $\sigma_k$ -Yamabe problem, and some aspects of the general theory and applications of fully nonlinear equations in conformal geometry.

**Mauro Carfora**, University of Pavia – *Ricci Flow in Theoretical and Mathematical Physics*

Ricci flow plays a natural role in many distinct physical theories. In this series of lectures we review a few case studies which characterize an important interaction between mathematics and physics. In particular, we shall discuss the relations between Ricci flow and non-linear sigma model theory, the applications of Ricci flow to general relativity, and finally some aspects of the role of Ricci flow in the theory of diffusion processes on Riemannian manifolds.

**Andrea Malchiodi**, SISSA, Trieste – *Functional Determinants and Conformal Geometry*

Conformal variations of renormalized determinants for covariant operators can be sometimes expressed by explicit formulas. It is natural then to find metrics which might extremize the determinants: for example in two dimensions this is a way to characterize constant curvature metrics. I will discuss the derivation of such formulas and some cases in which it is possible to find or characterize extremal metrics.

The course will consist of four lectures, during about one hour and a half each. The material covered in the lectures will be the following:

1. Review on the min–max formulas for eigenvalues of the Laplacian, Weyl’s asymptotic formula, heat kernels and the Minakshisundaram–Pleijel expansions.
2. Functional determinant of the Laplacian, Ray–Singer–Polyakov formulas, characterization of extremal metrics by Osgood–Phillips–Sarnak, Onofri inequality.
3. Isospectral surfaces, higher order heat invariants and compactness results for isospectral metrics in high order norms.
4. Functional determinants in higher dimensions, compactness of isospectral metrics, determinants of conformally invariant operators, existence and uniqueness of extremal metrics.

The course will require some basic knowledge and familiarity with Riemannian geometry, Sobolev spaces and regularity theory for elliptic partial differential equations.

**Jeff Viaclovsky**, University of Wisconsin, Madison – *Geometry of Anti–Self–Dual Metrics*

I will begin with some basic notions in 4–dimensional Riemannian geometry in order to define the concept of half–conformally–flat metrics (which are also known as self–dual or anti–self dual metrics), and a generalization of these known as Bach–flat metrics. These equations are elliptic in a suitable gauge, and I will discuss a basic regularity theorem. I will also discuss volume growth and Cheeger–Gromov convergence of such metrics, and discuss some explicit examples.

**Lei Ni**, University of California, San Diego – *Geometric Applications of Ricci Flow*

Ricci flow has been proven powerful in study the topology and geometry of manifolds by the fundamental work of Hamilton and Perelman. We shall focus on some recent progresses in applying Ricci flow to study the structure of Riemannian and Kähler manifolds.

**WorkGroup on Mean Curvature Flow**

Our aim is to organize an introductory but quite complete course on the results and techniques in the “classical” approach to mean curvature flow of hypersurfaces (the works of Ecker, Huisken, Hamilton, etc).

Possibly, we will try to cover in a self–contained way the analysis from the small time existence theorem to the asymptotic behavior at the first singular time.

A list of people collaborating to the workgroup will include Roberta Alessandroni, Charles

Baker, Giovanni Bellettini, Giovanni Catino, Zindine Djadli, Carlo Mantegazza, Annibale Magni, Lorenzo Mazzieri, Reto Müller, Matteo Novaga.

## 6. WORKSHOPS

### **Geometric Flows in Mathematics and Theoretical Physics – 22/25 June 2009 In collaboration with GNFM**

The workshop aims to report on the recent interdisciplinary developments about geometric flows with an emphasis on the discussion of problems and new research directions coming from the mathematical and theoretical physics communities.

Ricci flow has been the point of departure and the motivating example for important developments in geometric analysis, most spectacularly for G. Perelman's proof of Thurston's geometrization program for three-manifolds and of the famous Poincaré conjecture. As it is well known, the Ricci flow also appears naturally in Quantum Field Theory (QFT), as the weak coupling limit of the renormalization group analysis of nonlinear sigma models. The existence of a QFT avatar is by no means an exclusive of Ricci flow, but it is a property shared by the mean curvature flow, by the curve shortening flow, and many other (weakly) parabolic flows of geometrical nature. Such an observation suggested that methods of quantum field theory may have useful applications in geometric flow theory, and has been the motivating stimulus of a remarkable series of meetings among mathematicians and physicists working in geometric analysis and in quantum field theory. The first of such meetings was held at the Albert Einstein Institute in Potsdam in November 2006. That meeting was very successful and set an agenda that in a year span has already seen other two very stimulating workshops in Banff and Munich. Many ideas have been put forward during these meetings and it is becoming increasingly apparent that the interaction between mathematicians and physicists in this area is indeed very fruitful. The fourth in this series of workshops will be held in Pisa at the Centro De Giorgi of the Scuola Normale Superiore from the 22<sup>nd</sup> to the 25<sup>th</sup> of June 2009. As in the previous meetings, our objective is to provide a fast and informal channel of communications between the two communities, suggesting new problems of common interest and possible strategies of solutions. The properties of geometric flows discussed in depth by the methods of geometric analysis may shed light on the nature of significant models of QFT, whereas the imagination of QFT, often transcending and transgressing formal boundaries, may suggest new powerful strategies in geometric flow theory.

Speakers will include:

*M. Athanassenas, G. Bellettini, F. Costantino, D. Friedan, D. Glickenstein, R. Gulliver, J. Hoppe, A. Magni, S. Maillot, M. Petropoulos, M. Troyanov, V. Suneeta, M.-T. Wang, E. Woolgar*

For more information, see at <http://cvgmt.sns.it/MPRicci-Workshop>  
or contact Mauro Carfora at [Mauro.Carfora@pv.infn.it](mailto:Mauro.Carfora@pv.infn.it)

### **General Conference “GFO in Pisa” – 29 June/3 July 2009**

Speakers will include:

*R. Alessandroni, C. Baker, L. Bessières, G. Besson, F. Bonsante, E. Cabezas-Rivas, G. Catino, E. Hebey, T. Lamm, P. Le Floch, L. Martinazzi, L. Mazzieri, R. Müller, M. Novaga, M. Rupflin, H. Seshadri,*

*W. Sheng, M. Simon, C. Sinestrari, J. Streets, M. Struwe, G. Tarantello*

For more information, see at <http://cvgmt.sns.it/GFO-Conf>  
or contact Zindine Djadli at [Zindine.Djadli@ujf-grenoble.fr](mailto:Zindine.Djadli@ujf-grenoble.fr)

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