Centro di Ricerca Matematica "Ennio De Giorgi"

Intensive research period

Configuration Spaces: Geometry, Combinatorics and Topology

Algebra and Geometry of Configuration Spaces and related structures

June 21-25, 2010

All the lectures of this workshop will take place in Aula Dini.

Preliminary list of speakers:

Nero Budur: *The monodromy conjecture for hyperplane arrangements*

Jose Ignacio Cogolludo-Agustin: *Quasitoric decompositions of plane curves*

Daniel C. Cohen: *Some random complexes and associated spaces*

Emanuele Delucchi: *Complex Matroids*

David Garber: *Some properties of a conjugation-free geometric presentation of fundamental groups of arrangements*

Anthony Henderson: *Cohomology of real Coxeter toric varieties*

Olga Holtz: *Zonotopal algebra: the hierarchy*

Toshitake Kohno: *Quantum representations of mapping class groups and their images*

Gustav Lehrer: *Hodge numbers, rational points and discriminant varieties*

Anatoly Libgober: *Application of the Hodge theory to cohomology of local systems*

Eduard Looijenga: *Hyperbolic structures on toric arrangement complements*

Ivan Marin: *A Krammer representation for complex braid groups*

Daniel Matei: *Logarithmic sheaves and arrangements of hyperplanes*

Luca Moci: *A Tutte polynomial for toric arrangements*

Amos Ron: *Splines at the crossroads of algebra, combinatorics, geometry and numerical analysis, or Introduction to zonotopal algebra I*

Hal Schenck: *Equivariant Chow cohomology of nonsimplicial toric varieties*

John Shareshian: *Lower intervals in the Bruhat order and inversion arrangements*

Mina Teicher: *On Fundamental groups of complements of line arrangements*

Hiroaki Terao: *Combinatorial and algebro-geometric properties of free arrangements*

Alexander Varchenko: *Conformal blocks and equivariant cohomology*

Michèle Vergne: *Varchenko's analytic continuation of polytopes, and wall crossing formulae*

Michelle Wachs: *On the representation of the symmetric group on the cohomology of the toric variety associated with the type A Coxeter complex*

Miguel A. Xicotencatl: *On mapping class groups of non-orientable surfaces*

last update: June 21, 2010

Algebra and Geometry of Configuration Spaces and related structures					
June, 21-25, 2010					
Timetable					
Hour	Monday, 21st	Tuesday, 22nd	Wednesday, 23rd	Thursday, 24th	Friday, 25th
9:00-10:00	Registration	N. Budur	A. Ron	M. Wachs	D. Matei
10:00-10:30	Coffee break				
10:30-11:30	M. Vergne	I. Marin	O. Holtz	T. Kohno	D. Cohen
11:45-12:45	E. Delucchi	H. Terao	A. Varchenko	E. Looijenga	G. Lehrer
12:45-14:45	Lunch break				
14:45 -15:45	A. Libgober	M. Teicher		A. Henderson	
15:45-16:15	Coffee break			Coffee break	
16:15-17:15	J. Shareshian	H. Schenck		J. I. Cogolludo-Augustin	
17:30-18:00	L. Moci	D. Garber		M. A. Xicotencatl	

Abstracts:

Nero Budur List of speakers
University of Notre Dame
The monodromy conjecture for hyperplane arrangements

We present recent results about the local zeta functions, Bernstein-Sato polynomials, and the monodromy conjecture for hyperplane arrangements. This is joint work with various people: M. Mustata, M. Saito, Z. Teitler, and S. Yuzvinsky.

Jose Ignacio Cogolludo-Agustin List of speakers
Universidad De Zaragoza
Quasitoric decompositions of plane curves

Daniel C. Cohen List of speakers
Louisiana State University
Some random complexes and associated spaces

The theory of random graphs has been an integral branch of discrete mathematics for about 50 years. The study of random simplicial complexes of higher dimension is a much more recent development.
We discuss some results on random complexes, focusing mainly on low dimensional cases, and then pursue some implications for spaces associated to random complexes.

Emanuele Delucchi List of speakers
SUNY Binghamton
Complex Matroids

The combinatorial data associated to an arrangement is encoded by the associated matroid. In the case of real arrangements, this data can be refined by the structure of oriented matroids. In fact, matroid theory can be viewed as an abstraction of some combinatorial properties of linear dependencies among elements of vector spaces, whereas the theory of oriented matroids specifically deals with the combinatorics of linear dependencies over the real numbers. A substantial part of the richness of those theories lies in the fact that they each can be axiomatized in a number of equivalent ways. Some work has been devoted to the search for a similar structure for linear dependencies over the complex numbers.
After a quick review of matroids and oriented matroids, we will present our attempt at a theory of "complex matroids" that shares much of the structural richness of oriented matroid theory. This is joint work with Laura Anderson.

David Garber List of speakers
Holon Institute of Technology
Some properties of a conjugation-free geometric presentation of fundamental groups of arrangements

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A conjugation-free geometric presentation of a fundamental group is a presentation with the natural topological generators $x_1, \ldots, x_n$ and the cyclic relations:

$$x_{i_k} x_{i_{k-1}} \cdots x_{i_1} = x_{i_{k-1}} \cdots x_{i_1} x_{i_k} = \cdots = x_{i_1} x_{i_k} \cdots x_{i_2}$$

with no conjugations on the generators. We study some properties of this type of presentations for a fundamental group of a line arrangement's complement. We actually show that a large family of these presentations satisfy a completeness property in the sense of Dehornoy. The completeness property is a powerful property which leads to many nice properties concerning the presentation (as the left-cancellativity in the associated monoid and yields some simple criterion for the solvability of the word problem in the group). Joint work with Meital Eliyahu and Mina Teicher.

Anthony Henderson List of speakers
University of Sydney
Cohomology of real Coxeter toric varieties

There are several important classes of varieties X for which the (co)homology of the space of complex points $X(\mathbb{C})$ has some combinatorial or algebraic significance. Examples include toric varieties, complements of subspace arrangements, and moduli spaces of curves. As a general rule, it is harder to make such statements about the (co)homology of the space of real points $X(\mathbb{R})$, unless one takes coefficients in $\mathbb{Z}/2\mathbb{Z}$. A notable exception to this rule is the result of Etingof–Henriques–Kamnitzer–Rains on the rational cohomology of the real points of the moduli space of stable genus 0 curves with marked points. In this talk I will discuss this general context, and report on a project to describe the rational cohomology of $X(\mathbb{R})$ where X is the toric variety associated to the Coxeter fan of a Weyl group. This is joint work with Gus Lehrer.

Olga Holtz List of speakers
UC Berkeley
Zonotopal algebra: the hierarchy

I will discuss the hierarchy of zonotopal algebras between internal and external in more detail, including the central case (also known as Dahmen-Micchelli spaces).

Toshitake Kohno List of speakers
Graduate School of Mathematical Sciences, the University of Tokyo
Quantum representations of mapping class groups and their images

There is an action of the mapping class groups on the space of the conformal blocks for Riemann surfaces. This action defines so called quantum representation of mapping class groups. We give qualitative estimate for the images of such representations. In particular, we show that the image of any Johnson subgroup contains a non-abelian free group. We also give an answer to conjectures by Squier on Burau representations of braid groups. This is a joint work with Louis Funar.

Gustav Lehrer List of speakers
University of Sydney
Hodge numbers, rational points and discriminant varieties

Anatoly Libgober List of speakers
Univeristy of Illinois at Chicago
Application of the Hodge theory to cohomology of local systems

Eduard Looijenga List of speakers
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Hyperbolic structures on toric arrangement complements

Ivan Marin List of speakers
Institut de Mathématiques de Jussieu, Université Paris 7
A Krammer representation for complex braid groups

It is known by work of Krammer, Bigelow, Digne and Cohen-Wales, that the Artin groups of finite Coxeter type are linear. Furthermore, special properties of these groups can be obtained from the given faithful representation. A natural question is to ask whether the same thing holds for the more general case of complex braid groups, that is the 'braid groups' associated to arbitrary complex reflection groups and their reflection arrangement. We will present a natural monodromy representation for these groups which generalizes the Coxeter case, and investigate the consequences of its possible faithfulness.

Daniel Matei
Romanian Academy
Logarithmic sheaves and arrangements of hyperplanes

We discuss the sheaves of logarithmic one-forms introduced recently by I. Dolgachev. To a divisor on a smooth variety is associated a certain sub-sheaf of the sheaf of logarithmic one forms considered by K.Saito. The latter sheaf coincides with the double dual of the former. In the particular case where the divisor is an arrangement of hyperplanes in the projective space, the sub-sheaf turns out to be a Steiner sheaf, possesing a certain type of resolution. M. Kapranov and I. Dolgachev, and later J. Valles, studied the case of generic arrangements, for which the sheaves in question are in fact locally free. They proved that two arrangements with isomorphic bundles of logarithmic one-forms coincide unless they osculate a normal rational curve. Motivated by a conjecture of I. Dolgachev, we address here the similar problem for arbitrary hyperplane arrangements, that is to what extent the above Steiner logarithmic sheaf determines the arrangement. This is a joint work with D. Faenzi and J. Valles.

Luca Moci
Università di Roma Tre
A Tutte polynomial for toric arrangements

A toric arrangement is a finite family of hypersurfaces in a torus, every hypersurface being the kernel of a character. We describe some properties of such arrangements, and we compare them with hyperplane arrangements. The Tutte polinomial is an invariant which encodes a rich description of the topology and the combinatorics of a hyperplane arrangement, and satisfies a simple recurrence. We introduce the analogue of this polynomial for a toric arrangement. Furthermore, we show that our polynomial computes the volume of the related zonotope, counts its integral points, and provides the graded dimension of a space of quasipolynomials introduced by Dahmen and Micchelli to study partition functions.

Amos Ron List of speakers
University of Wisconsin-Madison
Splines at the crossroads of algebra, combinatorics, geometry and numerical analysis, or Introduction to zonotopal algebra I

Splines are compactly supported piecewise analytic functions defined on the Euclidean domain $\mathbb{R}^n$, $n \geq 1$. In my talk, I will describe connections between spline theory on the one hand, and various topics, setups and problems in algebra, in combinatorics, in geometry and in numerical analysis.
A partial list of areas and topics that are pertinent (not all discussed...) is the following:

1. Numerical analysis (e.g., multivariate polynomial interpolation)
2. Graph theory (e.g., Tutte polynomial and parking functions)
3. Enumerative combinatorics (e.g., f-vectors of a matroid or magic square problems)
4. Convex geometry and lattice geometry (e.g., hyperplane arrangement and zonotopes).
5. Non-commutative algebra and group representation (e.g., partition functions, Weyl's formula and Macdonald polynomials)
6. Commutative algebra, algebraic geometry and homological algebra (e.g., power ideals, toric varieties, and (equivariant) cohomology of hyperplane arrangements)

The actual talk contains fours parts. The 1^{st} is an elboration on specific nature of the interface between spline theory and algebra, combinatorics and geoemtry. The 2^{nd} is quick histrorical survey of zonotopal algebra. The 3^{rd} introduces the the least map and its basic properties. This map was borne out of the attempt to provide a universal and canonical solution to the problem of polynomial interpolation at arbitrary finite pointset of $\mathbb{R}^n$. However, I will use it in my talk for a different purpose: the map connects between geometry and algebra, hence can be used to validate the coherence in our constructions and theories. The 4^{th} part consists of examples within the zonotopal algebra framework that illustrates this point.

Hal Schenck List of speakers
University of Illinois
Equivariant Chow cohomology of nonsimplicial toric varieties

John Shareshian List of speakers
Washington University in St. Louis
Lower intervals in the Bruhat order and inversion arrangements

In joint work with A. Hultman, S. Linusson and J. Sjöstrand, we proved the following conjecture of A. Postnikov. Let w be an element of the symmetric group S_n. Let $\mathcal{A}_w$ be the arrangement of those hyperplanes in $\mathbb{R}^n$ defined by the equations $x_i = x_j$ whenever (i, j) is an inversion of w. Then

(1) The number of regions in the complement of $\mathcal{A}_w$ is at most the number of elements below w in the Bruhat order, and

(2) equality holds in (1) if and only if w avoids a fixed (finite and known) set of patterns.

I will describe in some detail our proof of (1), which works for any finite Coxeter group. I will discuss some aspects of (2) if time permits.

Mina Teicher List of speakers
Bar-Ilan University
On Fundamental groups of complements of line arrangements

Hiroaki Terao List of speakers
Department of Mathematics, Hokkaido University
Combinatorial and algebro-geometric properties of free arrangements

Alexander Varchenko List of speakers
University of North Carolina at Chapel Hill
Conformal blocks and equivariant cohomology

Michèle Vergne List of speakers
Centre National de la Recherche Scientifique Ecole Polytechnique. Centre de Mathematiques Laurent Schwartz
Varchenko's analytic continuation of polytopes, and wall crossing formulae

Following Varchenko, we define a set theoretic "analytic" continuation of a simple polytope. We study the continuity properties of the deformation. We describe a set theoretic jump and we relate it to Paradan's formula for jumps of partition functions.

Michelle Wachs List of speakers
University of Miami
On the representation of the symmetric group on the cohomology of the toric variety associated with the type A Coxeter complex

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We give a decomposition of the representation of the symmetric group on the cohomology of the toric variety associated with the type A Coxeter complex into virtual representations, which we conjecture are actual representations. Since this decomposition has significance in enumerative combinatorics, we are hopeful that it might have some geometric significance as well. This is joint work with John Shareshian.

Miguel A. Xicotencatl List of speakers
CINVESTAV
On mapping class groups of non-orientable surfaces

The mapping class group (m.c.g.) Γ_g^+ of an oriented surface M_g, is the group of isotopy classes of orientation preserving diffeomorphisms of M_g. Variations of this group include the full m.c.g. $\Gamma_g^{\pm}$, and the m.c.g. with marked points. Recently, the study of m.c.g.'s has also been extended to the non-orientable case. When considering all diffeomorphims, one shows the m.c.g. of a surface M (oriented or not) with k marked points, surjects onto the full m.c.g. $\Gamma(M)$ with kernel $\tilde{\Gamma}^k(M)$. Moreover, using configuration spaces we construct a space with fundamental group $\tilde{\Gamma}^k(M)$. In the case when M is the projective plane or the Klein bottle, we use these spaces to study the cohomology of $\tilde{\Gamma}^k(M)$ and groups related to the braid groups of M. This is joint work with Miguel A. Maldonado.

Location

All the lectures of this workshop will take place in Aula Dini, inside Palazzo del Castelletto (number 1 in the map below, look also at `http://www.crm.sns.it/hpp/practical/maps.html`).

Table 1: Map with the location of Palazzo del Castelletto (number 1)