

**Centro di Ricerca Matematica “Ennio De Giorgi”**  
Intensive research period  
**Configuration Spaces: Geometry, Combinatorics and Topology**

## **Combinatorial and geometric aspects of Hyperplane Arrangements**

May 24-26, 2010

All the lectures of this workshop will take place in Aula Dini.

### **Preliminary list of speakers:**

**Alejandro Adem:** *Homotopy theory and spaces of representations*

**Enrique Artal Bartolo:** *Characteristic varieties of quasiprojective groups*

**Eric Babson:** *Families of projective covers of path algebra representations*

**Ruth Charney:** *Automorphisms of right-angled Artin groups*

**Graham Denham:** *Enumerative problems for logarithmic forms on hyperplane complements*

**Alexandru Dimca:** *Hodge Theory and Hyperplane arrangements*

**Michael Falk:** *Brunnian braids and homomorphisms of arrangement groups*

**Samuel Gitler:** *Infinite families of simple polytopes and of simplicial complexes with applications to toric topology*

**Stefan Papadima:** *An explicit Kontsevich integral for welded braids*

**Luis Paris:** *Minimal index of the mapping class groups*

**Bernd Sturmfels:** *Mustafin Varieties*

**Alexandru Suciu:** *Free abelian covers and arrangements of Schubert varieties*

**Ulrike Tillmann :** *From configuration spaces to moduli space - from braid to mapping class groups*

**Volkmar Welker:** *Geometric Combinatorics of Weyl Groupoids*

**Masahiko Yoshinaga:** *Minimal stratification for line arrangements*

*last update: May 19, 2010*

## Combinatorial and geometric aspects of Hyperplane Arrangements

May 24-26, 2010

### Timetable

| Hour          | Monday, May 24th | Tuesday, May 25th | Wednesday, May 26th |
|---------------|------------------|-------------------|---------------------|
| 9:00 - 10:00  | Registration     | A. Dimca          | A. Adem             |
| 10:00 - 10:30 |                  | Coffee break      |                     |
| 10:30 - 11:30 | U. Tillmann      | E. Babson         | S. Gitler           |
| 11:45 - 12:45 | M. Falk          | M. Yoshinaga      | G. Denham           |
| 12:45 - 14:45 |                  | Lunch break       |                     |
| 14:45 - 15:45 | B. Sturmfels     | A. Suciu          | S. Papadima         |
| 15:45-16:15   |                  | Coffee break      |                     |
| 16:15 - 17:15 | V. Welker        | L. Paris          | R. Charney          |
| 17:30 - 18:30 | E. Artal Bartolo |                   |                     |

## Abstracts:

### Alejandro Adem

University of British Columbia

*Homotopy theory and spaces of representations*

LIST OF SPEAKERS

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Using spaces of homomorphisms and the descending central series of the free groups, simplicial spaces are constructed for each integer  $q > 1$  and every topological group  $G$ , with realizations  $B(q, G)$  that filter the classifying space  $BG$ . In particular for  $q = 2$  this yields a single space  $B(2, G)$  assembled from all the n-tuples of commuting elements in  $G$ . Homotopy properties of the  $B(q, G)$  will be described for finite groups, and cohomology calculations provided for compact Lie groups. Recent results on understanding both the number and stable homotopy type of the components of related spaces of representations will also be discussed.

### Enrique Artal Bartolo

Universidad de Zaragoza

*Characteristic varieties of quasiprojective groups*

LIST OF SPEAKERS

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Following the work of Beauville, Arapura, Delzant, Simpson a description of characteristic varieties of quasiprojective groups is given. This invariant measures the jumping numbers for the cohomology with respect to local system of coefficients. This description provides a number of obstructions (following ideas of Dimca, Papadima an Suciu) which can be applied to study the quasiprojectivity of Tits-Artin groups. It is a joint work with J.I. Cogolludo and D. Matei.

### Eric Babson

UC Davis

*Families of projective covers of path algebra representations*

LIST OF SPEAKERS

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We mark representations of the path algebra of a finite directed graph by a projective cover and consider the modules annihilated by the edge ideal. The resulting varieties are paved by fairly simple objects, and much of the geometry is carried by a 1-dimensional skeleton.

Joint work with Birge Huisgen-Zimmermann and Rekha Thomas.

**Ruth Charney**

Brandeis University

*Automorphisms of right-angled Artin groups*

LIST OF SPEAKERS

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Automorphism groups of free groups have many properties in common with automorphism groups of free abelian groups, i.e.,  $GL(n, \mathbb{Z})$ . Interpolating between these are the automorphism groups of right-angled Artin groups. We discuss the current state of knowledge about these groups.

**Graham Denham**

University of Western Ontario

*Enumerative problems for logarithmic forms on hyperplane complements*

LIST OF SPEAKERS

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The study of modules of logarithmic vector fields and forms on hyperplane complements now has a 30-year history that has revealed some interesting subtleties. For example, a famous formula of Solomon and Terao expresses the characteristic polynomial of the arrangement (matroid) in terms of a specialization of the Hilbert series of modules of logarithmic differentials: however, the Hilbert series of such a module is not uniquely determined by the matroid. I will describe some recent results that give new relations amongst the Chern classes of sheaves of logarithmic forms, in some cases leading to explicit formulas and a "geometric" explanation of Solomon and Terao's formula. This is joint work with Mathias Schulze.

**Alexandru Dimca**

Université de Nice Sophia Antipolis, France

*Hodge Theory and Hyperplane arrangements*

LIST OF SPEAKERS

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It was known since a long time that the cohomology of a hyperplane arrangement complement enjoys very special Hodge theoretic properties, which lead to a precise relationship between the associated characteristic varieties and the resonance varieties. After recalling in detail these results we discuss several recent extensions, covering in particular the case of Milnor fibers of hyperplane arrangements.

**Michael Falk**

Northern Arizona University

*Brunnian braids and homomorphisms of arrangement groups*

LIST OF SPEAKERS

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We describe a generalization of Stanford's theorem characterizing Brunnian braids, and give an application to fundamental groups of complements of complex hyperplane arrangements. In some cases one is able to show these arrangement groups are subdirect products of free groups, and draw conclusions about their residual properties and homological finiteness type. This is joint work with Daniel C. Cohen and Richard Randell.

**Samuel Gitler**

LIST OF SPEAKERS

Centro de Investigación y de Estudios Avanzados (Cinvestav) del Instituto Politécnico Nacional (IPN), Mexico

*Infinite families of simple polytopes and of simplicial complexes with applications to toric topology*

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We construct infinite families of simple polytopes associated to a given one as well as infinite families of abstract simplicial complexes associated with a given one. We will give some applications to toric topology.

**Stefan Papadima**

LIST OF SPEAKERS

Institute of Mathematics “Simion Stoilow” of the Romanian Academy,  
Bucharest

*An explicit Kontsevich integral for welded braids*

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Drinfeld used associators to construct faithful representations of braid groups into complete algebras of chord diagrams, whose extension to the group ring induces a multiplicative isomorphism, at the associated graded level. We do the same, for welded braid groups and oriented chord diagrams. Our representations are defined over the rationals, by simple explicit formulae. This is joint work with Barbu Berceanu.

**Luis Paris**

LIST OF SPEAKERS

Université de Bourgogne

*Minimal index of the mapping class groups*

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The minimal index of a group  $G$  is defined to be the minimal index for a proper subgroup of  $G$ . This invariant is not so exiting in general, except for groups such as the mapping class groups that are perfect and residually finite. The goal of this talk is to present a joint work with J. A. Barrick and V. Gebhardt where we calculate the minimal indices of the mapping class groups.

**Bernd Sturmfels**  
UC Berkeley  
*Mustafin Varieties*

LIST OF SPEAKERS

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A Mustafin variety is a degeneration of projective space induced by a point configuration in a Bruhat-Tits building. The special fiber is reduced and Cohen-Macaulay, and its irreducible components form interesting combinatorial patterns. For configurations that lie in one apartment, these patterns are regular mixed subdivisions of scaled simplices, and the Mustafin variety is a twisted Veronese variety built from such a subdivision. This connects our study to tropical and toric geometry. For general configurations, the irreducible components of the special fiber are rational varieties, and any blow-up of projective space along a linear subspace arrangement can arise. A detailed study of Mustafin varieties is undertaken for configurations in the Bruhat-Tits tree of  $PGL(2)$  and in the two-dimensional building of  $PGL(3)$ . The latter yields the classification of Mustafin triangles into 38 combinatorial types. This is joint work with Dustin Cartwright, Mathias Haebich and Annette Werner.

A pdf file of the talk is available on the arXiv at the web-page <http://front.math.ucdavis.edu/1002.1418>.

**Alexandru Suciu**  
Northeastern University, Boston  
*Free abelian covers and arrangements of Schubert varieties*

LIST OF SPEAKERS

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The connected, regular,  $\mathbb{Z}^k$ -covers of a finite CW-complex  $X$  are parametrized by the points in the Grassmannian of  $k$ -planes in  $V = H^1(X, \mathbb{Q})$ . Moving about this rational Grassmannian, and recording when the Betti numbers (up to some fixed degree  $i$ ) of the corresponding covers are finite carves out certain subsets  $\Omega_k^i(X)$  of  $\text{Gr}_k(V)$ .

In this talk, I will present a method for determining these sets, using the cohomology jumping loci of  $X$ , and the incidence correspondence between projective varieties and subvarieties of the Grassmannian. Under favorable conditions, the  $\Omega$ -invariants are controlled by certain arrangements of special Schubert varieties, which can be computed directly from the cohomology ring of  $X$ .

**Ulrike Tillmann**  
University of Oxford  
*From configuration spaces to moduli space - from braid to mapping class groups*

LIST OF SPEAKERS

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There are naturally defined homomorphisms between braid groups, mapping class groups, symmetric groups and automorphisms of free groups, mainly coming from geometry. We will describe these maps in (stable) homology.

**Volkmar Welker**

Philipps-Universität Marburg

*Geometric Combinatorics of Weyl Groupoids*

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LIST OF SPEAKERS

We study geometric objects associated to (finite) Weyl groupoids. Weyl groupoids arose from the work of V. Kac in his work on Lie superalgebras and independently in the study of Nichols algebras. Weyl groupoids include the classical (finite) Weyl groups as an important but very special case.

We are interested in the weak order, the Coxeter complex and the hyperplane arrangement associated to a Weyl groupoid. For Weyl groups or more generally Coxeter groups it is known that all these objects exhibit very nice behavior. We show which properties extend from the classical to the Weyl groupoid case and give examples when the extension fails.

This is joint work with I. Heckenberger

**Masahiko Yoshinaga**

Kyoto University

*Minimal stratification for line arrangements*

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LIST OF SPEAKERS

Several types of minimal presentations of  $\pi_1$  for arrangements complements have been known since long time. More generally, arrangements complements are known to be homotopy equivalent to minimal CW complex. In this talk, we focus on two dimensional complexified real arrangements and introduce “minimal stratification” which can be considered as a dual notion to minimal CW complex.

As applications, we will give a new minimal positive homogeneous presentation for the fundamental group, and discuss how to recover local system cohomology groups from the Aomoto complex.

## Location

All the lectures of this workshop will take place in Aula Dini, inside Palazzo del Castelletto (number 1 in the map below, look also at <http://www.crm.sns.it/hpp/practical/maps.html>).



Table 1: Map with the location of Palazzo del Castelletto (number 1)