

Centro di Ricerca Matematica “Ennio De Giorgi”  
Intensive research period  
Configuration Spaces: Geometry, Combinatorics and Topology

Young researchers’ seminars  
May-June, 2010

**Preliminary list of talks:**

- Đorđe Baralić:** *Topological obstructions for totally skew embeddings*  
MAY 13TH, 15:45-16:45, SALA CONFERENZE
- Priyavrat C. Deshpande:** *Homotopy theoretic techniques in arrangements (1)*  
MAY 13TH, 17:00-18:00, SALA CONFERENZE
- Priyavrat C. Deshpande:** *Homotopy theoretic techniques in arrangements (2)*  
MAY 21ST, 17:00-18:00, SALA CONFERENZE
- Kris Williams:** *Hyperplane Arrangements and the Milnor Fiber*  
MAY 21ST, 18:00-19:00, SALA CONFERENZE
- Denis Ibadula:** *The explicit forms of the Igusa local zeta functions of certain plane curves*  
MAY 27TH, 17:00-18:00, SALA CONFERENZE
- Jose Manuel Gomez:** *Stable splittings, spaces of representations and almost commuting elements in Lie groups*  
MAY 27TH, 18:00-19:00, SALA CONFERENZE
- Giacomo D’Antonio:** *Symmetric group actions on the cohomology of configuration spaces*  
MAY 28TH, 17:00-18:00, SALA CONFERENZE
- Filippo Callegaro:** *Homology of complex braid groups*  
MAY 28TH, 18:00-19:00, SALA CONFERENZE
- Max Wakefield:** *Derivation Radical Subspace arrangements*  
JUNE 3DR, 17:00-18:00, SALA CONFERENZE
- Matthew Miller:** *Lattices, hypergraphs, and the topology of subspace arrangements*  
JUNE 3DR, 18:00-19:00, SALA CONFERENZE
- Priyavrat C. Deshpande:** *Arrangement of Submanifolds and the Tangent - Bundle Complement*  
JUNE 10TH, 17:00-18:00, SALA CONFERENZE
- Mehdi Garrounian:** *Multiarrangements and Critical Points*  
JUNE 10TH, 18:00-19:00, SALA CONFERENZE
- Luca Moci:** *TBA*  
JUNE 18TH, 17:00-18:00, SALA CONFERENZE
- Bruno Benedetti:** *Collapse depth of triangulated manifolds*  
JUNE 18TH, 18:00-19:00, SALA CONFERENZE

*last update: June 4, 2010*

☞ Before the 17:00 seminars there will be a coffee break starting at 16:45.

## Abstracts:

### **Đorđe Baralić**

LIST OF TALKS

Mathematical Institute SANU, Beograd Serbia  
*Topological obstructions for totally skew embeddings*  
(May 13th, 15:45-16:45, Sala Conferenze)

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There are two sub-themes (reflecting the work I have been doing from my PhD thesis) that more precisely describe what I would like to talk about:

- (a) Equivariant methods and equipartitions of measures by hyperplanes;
- (b) Skew embeddings of manifolds.

### **Priyavrat C. Deshpande**

LIST OF TALKS

University of Western Ontario  
*Homotopy theoretic techniques in arrangements*  
(May 13th and May 21st, 17:00-18:00, Sala Conferenze)

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One of the important aspect of hyperplane (or subspace) arrangement is to understand how far the topology of the complement of a union of hyperplanes (subspaces) is determined by the combinatorics of the arrangement. In recent years a lot of techniques from homotopy theory, notably the homotopy colimits have been used to discover some connections between combinatorics and topology. Homotopy colimit is an important idea originating in homotopy theory, that was developed by Quillen, Bousfield, Kan and others. It has not only reached remarkable extension and depth but it has also proved to be a versatile tool in a lot of other areas of mathematics.

The aim of my first talk is to motivate and explain the construction of homotopy colimits and how it is used to understand the homotopy type of the arrangement complement. I will do it using examples and pictures avoiding technical jargon from category theory. In my second talk I will describe how the Bousfield-Kan spectral sequence is used to compute the (co)homology of the complement. If time permits I will also discuss the fundamental group of a homotopy colimit. These talks are based on the works of Welker-Ziegler-Zivaljevic 1999, Delucci 2006 and Dror Farjoun 2004 and some classical material on model categories.

**Kris Williams**

University of Iowa

*Hyperplane Arrangements and the Milnor Fiber*

*(May 21st, 18:00-19:00, Sala Conferenze)*

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LIST OF TALKS

The complement of a central complex arrangement is a fiber bundle over  $\mathbb{C}^*$  with fiber  $F$ , the Milnor fiber. The fiber can also be realized as the cyclic  $d$ -fold covering space of the decone of the arrangement. Several authors have studied the Milnor fiber in this context, finding various restrictions on the possible homology groups. Using local system homology and covering space theory, we will explore conditions on the decone of 3-arrangements that have strong implications on the first betti number of the Milnor fiber as well helping to eliminate the possibility of torsion in the first homology group.

**Denis Ibadula**

Universitatea "Ovidius" Constanta Romania

*The explicit forms of the Igusa local zeta functions of certain plane curves*

*(May 27th, 17:00-18:00, Sala Conferenze)*

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LIST OF TALKS

We explore the Igusa local zeta functions associated to the nondegenerate homogeneous polynomials of degree three in two variables (plane cubics) over  $\mathbb{Q}_p$ , for  $p \neq 2, 3$ . We identify two plane cubics  $F$  and  $F'$  if there exists a matrix  $g \in GL_2(\mathbb{Q}_p)$  such that  $(g \cdot F')(x_1, x_2) = F(x_1, x_2)$ , where  $(g \cdot F')(x_1, x_2) = F'((x_1, x_2)g^t)$  and  $g^t$  denotes the transposed of the matrix  $g$ . Hence, the isomorphism classes are orbits of  $GL_2(\mathbb{Q}_p)$ -action.

First, we determine explicitly the representative nondegenerate plane cubics over  $\mathbb{Q}_p$  of the orbits of  $GL_2(\mathbb{Q}_p)$ -action. Then, we explore the Igusa local zeta functions of the  $GL_2(\mathbb{Q}_p)$ -orbit of each representative. We prove that it suffices to reduce the investigation of Igusa local zeta functions of the  $GL_2(\mathbb{Q}_p)$ -orbit of any plane cubic  $F$  to the  $GL_2(\mathbb{Q}_p) \bmod \mathbb{Q}_p^\times GL_2(\mathbb{Z}_p)$ -orbit of  $F$ . Thus, the idea is to use the tree  $X := GL_2(\mathbb{Q}_p) / \mathbb{Q}_p^\times GL_2(\mathbb{Z}_p)$  and to write an arbitrary plane cubic  $F$  as  $g \cdot F_i$ , with  $g \in GL_2(\mathbb{Q}_p)$  and  $F_i$  one of the representatives of the orbits of the  $GL_2(\mathbb{Q}_p)$ -action. The key idea in our determination is to calculate  $Z_{F_i^g}$ , where  $g$  runs through a set of representatives of  $X$  and where  $F_i^g$  denote the primitive polynomial associated to  $g \cdot F_i$ .

**Jose Manuel Gomez**

LIST OF TALKS

University of British Columbia

*Stable splittings, spaces of representations and almost commuting elements in Lie groups*

*(May 27th, 18:00-19:00, Sala Conferenze)*

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Two elements  $x, y$  in a Lie group  $G$  are said to be almost commutative if  $[x, y]$  is in the center of  $G$ . The space of almost commuting elements is a good way to approach the space of commuting elements in  $G$ , a space of current interest. In this talk I will explain how the space of almost commuting elements in a compact Lie group splits after one suspension. This is a joint work with Alejandro Adem and Fred Cohen.

**Giacomo D'Antonio**

LIST OF TALKS

University of Bremen

*Symmetric group actions on the cohomology of configuration spaces*

*(May 28th, 17:00-18:00, Sala Conferenze)*

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The symmetric group  $S_n$  acts in a natural way on the configuration space  $F(M, n)$  of a manifold  $M$  and thus on its cohomology algebra  $H^*(F(M, n); \mathbb{C})$ . We will approach this topic with representation-theoretic methods, focusing on the cases  $M = \mathbb{R}^k$  and  $M = S^k$  of euclidean spaces and spheres. The first case can be worked out introducing an auxiliary extended action of  $S_{n+1}$  on the algebra  $H^*(F(\mathbb{R}^k, n); \mathbb{C})$ , thus allowing some sort of recursive argument. This extended action turns out to be one of the building blocks of the action of  $S_{n+1}$  on the cohomology  $H^*(F(S^k, n+1); \mathbb{C})$  of the configuration space of spheres.

**Filippo Callegaro**

LIST OF TALKS

Scuola Normale Superiore di Pisa

*Homology of complex braid groups*

*(May 28th, 18:00-19:00, Sala Conferenze)*

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I'll present some results of a work in progress in collaboration with Ivan Marin (Paris). We studied the braid groups associated to the reflection arrangements for complex reflection groups. For Coxeter groups there is a 1-to-1 correspondence with Artin groups. With complex reflection groups it happens that different reflection groups can give the same braid group and it is not always clear whether two complex braid groups are isomorphic. Homology can help to distinguish some of them, in particular for the infinite family of type  $B(de, e, r)$  and we can prove some isomorphisms too, but still open problems remain.

**Max Wakefield**

LIST OF TALKS

United States Naval Academy

*Derivation Radical Subspace arrangements*

*(June 3dr, 17:00-18:00, Sala Conferenze)*

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If a subspace arrangement contains a subspace of codimension higher than one then the associated logarithmic module of derivations is not free. In this lecture we discuss a possible generalization of freeness to subspace arrangements which we call derivation radical. We show that some special collections of subspaces of the braid arrangement are derivation radical. If time permits we will discuss how this definition relates to free hyperplane arrangements.

**Matthew Miller**

LIST OF TALKS

Bucknell University

*Lattices, hypergraphs, and the topology of subspace arrangements*

*(June 3dr, 18:00-19:00, Sala Conferenze)*

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In the study of subspace arrangements a common technique is to find a suitable combinatorial structure to model the behavior of the arrangement and then use the combinatorics to prove theorems. In this talk we will describe the intersection lattice of an arrangement and edge-colored hypergraphs as well as how they are used to study the topology of the complements of certain subspace arrangements. We will conclude with some recent results of mine and some joint work with Max Wakefield.

**Priyavrat C. Deshpande**

LIST OF TALKS

University of Western Ontario

*Arrangement of Submanifolds and the Tangent-Bundle Complement*

*(June 10th, 17:00-18:00, Sala Conferenze)*

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A real arrangement of hyperplanes is a collection of finitely many hyperplanes in a real vector space. It is known that the combinatorics of the intersections of these hyperplanes contains substantial information about the topology of the complement of the hyperplanes in the real as well as complexified space. For example, the cohomology of the complexified complement can be expressed in terms of the intersection lattice associated with the arrangement. The face poset of an arrangement defines a cell complex (the Salvetti complex) which has the homotopy type of this complement. In the same spirit, I will define the notion of an arrangement of submanifolds and its complexification. The goal is to investigate whether the combinatorics of the intersections of these submanifolds offer insight about the topology of the 'real' complement and the tangent bundle complement, an analogue of the complexified complement. The aim of this talk is to introduce these notions and report recent developments that generalize the theory of (real) hyperplane arrangements.

**Mehdi Garrounian**

University of Western Ontario

*Multiarrangements and Critical Points*

*(June 10th, 18:00-19:00, Sala Conferenze)*

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LIST OF TALKS

A multiarrangements is a natural generalization of hyperplane arrangements where we associate multiplicities to each hyperplane. The aim of this talk is to give a generalization of some parts of the theory of critical points of simple arrangements to multiarrangements. The main object is the logarithmic ideal of a multiarrangement which characterizes the freeness of the multiarrangements similar to the simple case. We will also explain the geometry of the zero locus of this ideal and present a complex which turns to be free resolution in case of free multiarrangements.

**Luca Moci**

Università di Roma Tre

*TBA*

*(June 18th, 17:00-18:00, Sala Conferenze)*

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LIST OF TALKS

**Bruno Benedetti**

TU Berlin

*Collapse depth of triangulated manifolds*

*(June 18th, 18:00-19:00, Sala Conferenze)*

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LIST OF TALKS

The “collapse depth” of a triangulated manifold  $M$  counts how many dimensions down we can get by collapsing  $M$  minus a facet. We compare this notion with classical concepts in discrete geometry (e.g. shellability), topology (e.g. knotted spheres), algebra (Cohen-Macaulayness) and enumerative combinatorics (e.g. locally constructible manifolds).

## Location

All the seminars will take place in Sala Conferenze, inside Collegio Puteano (number 2 in the map below) or in Aula Dini, inside Palazzo del Castelletto (number 1 in the map below). Look also at <http://www.crm.sns.it/hpp/practical/maps.html>.



Table 1: Map with the location of Palazzo del Castelletto (number 1) and Collegio Puteano (number 2)