# Financial Markets: Mathematical, Statistical, and Economic Analysis. April 29 - July 12, 2002

Today's financial markets require the use of ever more sophisticated mathematical techniques. From the 1970's advanced mathematics has had an enormous impact of the analysis of financial markets. This has engendered a prolific exchange between the world of economics and finance and the world of mathematics. This has given life to a new area of research, has given birth to new professional avenues (e.g. the mathematics of finance), and has transformed other fields (the area of insurance comes to mind where the concept of “risk transfer" has become central).

Financial models are the result of a combination of refined analyses involving mathematics, statistics, and economics. In fact, to produce a a valid financial model the mathematical counterpart must be validated based on concrete data furnished by statistics and guided by well established economic analysis.

This research trimester was organized by the Center for Mathematical Research "Ennio De Giorgi", recently founded at Scuola Normale Superiore, by the three University institutions of Pisa: the University of Pisa, the Scuola Normale Superiore, the Scuola Superiore di Studi Universitari e Perfezionamento S.Anna: the primary goal of this Trimester was to provide a state of the art description accessible both to young researchers wishing to do research in this area and to scientists from Mathematics, Economics and other disciplines interested in learning some of the most recent developments in the field.

Broad areas that were covered include: derivatives pricing and hedging in incomplete markets, term structure models, risk management, modelling dependence in Finance, Time Series analysis especially with reference to high frequency data, "artificial market models", aggregation issues and aspects of behavioural finance.

Two Committees were designated: a Scientific Committee, who had as main task to select the invited speakers, and a Local Organizing Committee who took care of the organization and of all the practical sides of the trimester.

The Scientific Committee was formed by professors: William Brock (University of Wisconsin, Madison), Darrel Duffie (Stanford University), Paul Embrechts (ETH, Zurich), Doyne Farmer (Santa Fè Institute), Hans Follmer (Humboldt University, Berlin).

The Local Organizing Committee was formed by professors Mariano Giaquinta and Giuseppe Da Prato (Scuola Normale Superiore), professor Maurizio Pratelli and dr. Paolo Guasoni (Università di Pisa), professor Giovanni Dosi and Dr. Giulio Bottazzi (Scuola Superiore S. Anna).

As main activity of the Trimester, several advanced courses (6-12 hours) were offered. The list of these courses is provided below, with a small presentation for each course. These courses were held at Scuola Normale, when the contents were mainly mathematical or statistical, otherwise they took place at Scuola Superiore S. Anna, were the contents were mainly related to economics.
All the lectures attracted many participants: in some of them, there were up to 80 people. Furthermore, professors Darrell Duffie and Nizar Touzi provided some notes on their

courses, which will be soon published as "Corsi della Scuola Normale Superiore".

Besides to these courses, some other lecturers were invited to give some research seminars by the Local Organizing Commitee.
In addition, all the participants were encouraged to present their research topics: several seminars were organized, and mainly held at the Mathematics Department of University of Pisa. The list of these seminars is provided below.

Several grants were available for young foreign researchers, who intended to attend the Trimester, with a minimum stay of 2 weeks: in the end, 17 people benefited by these grants (the list of these people is also provided). Italian researchers were refunded of their expenses, if they had not any other financial support.

All the participants were allowed to access to the libraries of Scuola Normale Superiore, Scuola Superiore S. Anna and Mathematics Department of University of Pisa.

In addition, at the Mathematics Department some rooms were available to the participants, to be used as offices.

# ADVANCED COURSES

**Kerry Back (Washington University in St. Louis)**
***ASYMMETRIC INFORMATION ON SECURITIES MARKET***

I will lecture on models of informed trading in securities markets - primarily the Kyle model but also the Glosten-Milgrom model and the relation between the two. As an introduction, I will describe the single-period Kyle model and its relation to rational expectations models. The majority of the lectures will concern the continuous-time Kyle model. This model is a model of filtering and control - filtering by market makers to discern the information in orders, and control by the informed trader or traders of their portfolio processes to maximize expected utility. The fundamental distinction between this model and the Merton model (or rational expectations models) is that the informed traders understand that their trades affect prices, and they consider the effect on prices when they trade. The key issues addressed are the rate at which the market learns the information of traders, and the depth of the market (the extent to which trades affect prices).

**Michel Dacorogna (Converium)**
***USING HIGH FREQUENCY DATA TO UNDERSTAND MARKET DYNAMICS***

**Darrell Duffie (Stanford University)**
***CREDIT RISK MODELING WITH AFFINE PROCESSES***

This course will begin with the definition and properties of affine processes, a class of Markov processes that is ideally suited to modeling credit risk and the valuation of defaultable bonds, among other financial applications. The course will emphasize applications to intensity-based counting processes for correlated defaults, corporate or sovereign term-structure modeling, credit-rating transition risk, credit default swap pricing, and the pricing and risk measurement of collaterized debt obligations. It is assumed that participants have basic knowledge of Markov processes and stochastic calculus. Familiarity with counting processes and conventional models of the term structure of interest rates will also be quite helpful, but not essential.

**Cars Hommes (CeNDEF, University of Amsterdam)**
***NONLINEAR ECONOMIC DYNAMICS AND HETEROGENUOUS BELIEFS***

This course discusses markets viewed as evolutionary adaptive systems. Agents are boundedly rational and choose investment strategies that have performed wellin the recent past. In these nonlinear evolutionary systems prices and beliefs co-evolve over time. A key issue is who will win such an evolutionary market competition. Will the rational agent drive out all other types, or can boundedly rational agents survive evolutionary competition even in the long run?
Theoretical work as well as laboratory experiments will be discussed.

**Literature:**


Brock, W.A., and Hommes, C.H., (1997), A rational route to randomness, Econometrica 65, 1059-1095.
Brock, W.A., and Hommes, C.H., (1998), Heterogeneous beliefs and routes to chaos in a simple asset pricingmodel, Journal of Economic Dynamics and Control 22, 1235-1274.
Brock, W.A., Hommes, C.H., and Wagener, F., (2002), Evolutionary dynamics in markets with many trader types, CeNDEF working paper, University of Amsterdam.
Hommes, C.H., (2001), Financial markets as nonlinear adaptive evolutionary systems, Quantitative Finance 1, 149-167.
Hommes, C.H., Sonnemans, J., Tuinstra, J. and van de Velden, H., (2002), Learning in cobweb experiments, CeNDEF working paper, University of Amsterdam.
Hommes, C.H., Sonnemans, J., Tuinstra, J. and van de Velden, H., (2002), Expectations and bubbles in asset pricing experiments, CeNDEF working paper, University of Amsterdam.


**Youri Kabanov (University of Franche-Comte, Besancon)**
***THEORY OF FINANCIAL MARKETS WITH TRANSACTION COSTS***

**Dilip Madan (University of Maryland)**
***LEVY PROCESSES IN FINANCIAL MODELING***

**Alexander McNeil (ETH Zurich)**
***QUANTITATIVE RISK MANAGEMENT***

Quantitative methodology is an increasingly important component of risk management in financial institutions. Financial risk management presents an extremely interesting area of application for statistics with many new challenges. Whereas much of traditional statistics concerns the average, the normal and the expected, risk management has more to do with the extreme, the abnormal and the unexpected. Central technical issues will be modelling the volatility of financial instruments, modelling extreme values and modelling dependent risks. We will examine methods relevant for both market and credit risk management.

**Provisional Contents:**

1. Basic concepts of risk management
2. Standard industry approaches
3. Multivariate risk models and copulas
4. Modelling financial time series
5. Extreme value theory
6. Applications to market and credit risk

**Thomas Mikosch (University of Copenhagen)**
***DEPENDENCE IN THE TAILS OF FINANCIAL TIME SERIES***

We aim at modeling the interplay between the tail behavior and the dependence structure of financial data. Log-returns of financial data often exhibit long range dependence effects (which affect the sample autocorrelation behavior of the absolute log-returns) and erratic behavior which results in heavy-tailed marginal distributions. In practice one often observes that financial log-return series can have infinite 3rd, 4th, 5th,... moments.
We consider too major classes of econometric models which try to capture the empirical behavior. Those include GARCH (generalized autoregressive conditionally heteroscedastic) and stochastic volatility processes. The GARCH case turns out to be a very complex one. We embed these processes in finite-dimensional stochastic recurrence equations.
Following classical work of Kesten, we show that the finite-dimensional distributions of these processes are multivariate regularly varying, thus they have infinite power moments of a certain degree. Heavy tails cause the classical limit theory for the sample autocorrelations to break down. Therefore one has to modify this asymptotic theory: unfamiliar infinite variance limit distributions occur and rates of convergence can be extremely slow. Point process techniques turn out to be important in this context.
The long range dependence effect of real-life data cannot be explained by GARCH models. A possible explanation for this effect is non-stationarity of the underlying time series. We will learn how statistical tools behave under non-stationarity and how they can fool us to see things which are not there.

**Marco Pagano (Università di Salerno)**
***MODELS OF MARKET MICROSTRUCTURE***

The course will provide an introduction to two main topics:

the determinants of liquidity in the secondary equity market, with applications to the role of transparency;
the link between secondary market liquidity and the primary equity market.

**References (main readings are starred)**


Amihud, Yakov, and Haim Mendelson, Asset Pricing and the Bid-Ask Spread, Journal of Financial Economics 17, 1986.
* Biais, Bruno, Larry Glosten and Chester S. Spatt, "The Microstructure of Stock Markets," CEPR Discussion Paper No. 3288, 2002.
Brennan, Michael J., and Avanidhar Subrahmanyam, Market Microstructure and Asset Pricing: On the Compensation for Illiquidity in Stock Returns, Journal of Financial Economics 41, 1996.
Ellul, Andrew and Marco Pagano, "IPO underpricing and after-market liquidity," unpublished manuscript, March 2002 (to be distributed on the spot).
* O'Hara, Maureen, Market Microstructure Theory, Basil Blackwell, Oxford, 1995
Pagano, Marco, The Changing Microstructure of European Equity Markets, in European Securities Markets: Implementing the Investment Services Directive and Beyond, edited by Guido Ferrarini, Kluwer Law International, 1997.
Pagano, Marco and Ailsa Röell, Auction and Dealership Markets: What is the Difference?, European Economic Review, Vol. 36, 2/3, April 1992.


**Kenneth Singleton (Stanford University)**
***ECONOMETRIC ANALYSIS OF DYNAMIC ASSET PRICING MODELS***

These lectures will be in three parts:

Generalized and simulated method of moments estimation of asset pricing models, with an application to modeling stochastic volatility;
Econometric analysis of dynamic term structure models;
Econometric analysis of dynamic option pricing models, with applications to equity options.

**Spyros Skouras (Santa Fe Institute)**
***THE ECONOMETRICS OF FINANCIAL DECISION MAKING***

This course will provide an overview of econometric methods used to making and understand financial decisions. I will review empirical evidence suggesting that standard econometric modelling methods perform poorly when the model is to be used for financial decision making. This fact, well-known to practitioners and a serious obstacle to the application of the theory of financial decision-making, is due to the particular nature of financial decisions. In particular, financial objectives are such that the performance of decisions is sensitive with respect to the specification of models for agent objectives, models for financial series and uncertainty about the parameters of both these models. Since it is impossible or at least prohibitively costly to be sufficiently accurate in the specification of these features of the decision environment, alternative approaches are in order. I review econometric methods that have been found useful in overcoming some of the difficulties with standard methods, emphasising the `decision-based approach to econometric modelling'. This approach works in terms of a slightly weaker performance criterion, is consistent with reasonable axioms on agent learning and nests the standard approach as a special case that should be used under (very) special conditions.

**Nizar Touzi (C.R.E.S.T.)**
***THE SUPER-REPLICATION PROBLEM UNDER PORTFOLIO CONSTRAINTS***

We present present an overview of the problem of super-replication under portfolio constraints. We start by examining the duality approach and its limitations. We then concentrate on the direct approach in the Markov case which allows to handle general large investor problems and gamma constraints. In the context of the Black and Scholes model, the main result from the practical view-point is the so-called face-lifting phenomenon of the payoff function.

**Outline:** 1. The duality approach
1.1. Dual formulation of the hedging problem
1.2. HJB equation from the dual formulation
1.3. Applications: The Black-Scholes model with portfolio constraints, the uncertain volatility model
1.4. Limitations of the duality approach: Large investor models, Gamma constraints
2. Dynamic programming on the initial formulation
2.1. The dynamic programming principle
2.2. HJB equation from the initial formulation
2.3. Application: hedging under gamma constraints

## SEMINARS:

**José Fajardo Barbachan (Catholic University of Brasìlia)** *Pricing Derivates on two levy-driven stocks*
**Francesca Biagini (University of Bologna)** *Minimal hedging for fractional brownian motion*
**Giulio Bottazzi (Scuola Superiore Sant'Anna, Pisa)** *Do we need heterogeneity? The*

*deterministic limit of a simple micro-model of financial market*

**Patrick Cheridito (ETH - Zurich)**

1. *Introduction to Fractional Brownian Motion*
2. *Arbitrage and Exclusion of Arbitrage in F.B.M. Models*

**Johanna Gaier(Technical University of Vienna)** *Asymptotic Ruin Probabilities and Optimal Investment*

**Stefan Geiss (Jyvaskyla University)**

1. *Is there a difference between equidistant andgeneral deterministic time nets in discrete time hedging?*
2. *On weak weighted BMO-spaces and the reason to consider them*

**Paolo Guasoni (University of Pisa)** *Optimal Investment with Transaction Costs and without Semimartingales*

**Jeroen Kerkhof (Tilburg University)** *Model Risk and Regulatory Capital*

**Marco Lippi (Università, La Sapienza, Roma)**

1. *Dynamic factor models: estimation and forecasting, part I*
2. *Dynamic factor models: estimation and forecasting, part II*

**Christopher Summer (Technical University of Vienna)** *Utility Maximization under Increasing Risk Aversion*

**Eva Strasser (Technical University of Vienna)** *The Supermartingale Property of Stochastic Integral with respect to a Local Martingale*

**Christopher Ting (Singapore Management University)** *Holding Period Returns and volatility of Nikkei*

**Astrid Van Landschoot (University of Ghent)** *How does a bank's risk profile influence the effect of credit risk? Evidence from bank stock returns*

**Jerzy Zabczyk (ICM and PAS, Warsaw)** *Markovian models for bond markets in discrete time*

**Gordan Zitkovic (Columbia University)** *Utility Maximization in Incomplete Semimartingale Markets with Random Endowment*

## LIST OF GRANTS

1. Barbachan José Fajardo
2. Carassus Laurence
3. Cheridito Patrick
4. Chung Hanse
5. Cudina Milica
6. Gaier Johanna
7. Grbac Zorana
8. Haboeck Ulrich
9. Kerkhof Jeroen
10. Krzysztof Tokarz
11. Onyango Silas
12. Ragea Valentin
13. Strasser Eva
14. Summer Christopher
15. Van Landschoot Astrid
16. Veza Tanja
17. Zitkovic Gordan