Centro di Ricerca Matematica Ennio De Giorgi
Intensive Research Period "Knots and Applications"
May 1, 2011 – July 31, 2011

Workshop

Geometric Topology of Knots

May 25–26, 2011

Schedule and Abstracts

Supported by
Istituto Nazionale di Alta Matematica "Francesco Severi"
Scuola di Dottorato "Galileo Galilei"
Università di Pisa

Schedule

	Wednesday, May 25	Thursday, May 26
9:00 – 9:45	Boileau	Luo
10:00 – 10:45	Lackenby	Purcell
10:45 – 11:15	*Coffee*	*Coffee*
11:15 – 12:00	Gordon	Schleimer
12:15 – 12:45	Charitos	Roukema
12:45 – 14:45	*Lunch*	*Lunch*
14:45 – 15:30	Tsvietkova	Vershinin
15:45 – 16:15	Pervova	Banks
16:15 – 16:45	*Coffee*	*Coffee*
16:45 – 17:15	Caillat-Gibert	Nechaev
17:30 – 18:00	Palumbo	Sayari

All activities will take place in
Aula Dini (Via del Castelletto 11, Pisa)

Abstracts

Jessica Banks (Oxford), *The Kakimizu complex of a link.*

We give an introduction to the Kakimizu complex of a link, covering a number of recent results. In particular we will see that the Kakimizu complex of a knot may be locally infinite, that the Alexander polynomial of an alternating link carries information about its Seifert surfaces, and that the Kakimizu complex of a special alternating link is understood.

Michel Boileau (Toulouse), *Commensurability classes of hyperbolic knot complements.*

We will discuss commensurability classes of hyperbolic knot complements. In the generic case of knots without hidden symmetries we describe the commensurability classes modulo the generalized Berge conjecture. In this case we show that knot complements which are commensurable are cyclically commensurable, and that there are at most 3 hyperbolic knot complements in such a commensurability class. Moreover if two hyperbolic knots without hidden symmetries have commensurable complements, then they are fibered with the same genus and chiral. This is a joint work with S. Boyer, R. Cebanu and G. Walsh.

Shanti Caillat-Gibert (Marseille), *Existence of taut foliations in Seifert fibered homology* 3*-spheres.*

We focus on the existence of taut foliations on compact 3-manifolds. David Gabai solved the problem for 3-manifolds with non-trivial homology, saying that they always admit a taut foliation. It remains the case of rational homology spheres. Here, we will study the case of Seifert rational homology spheres. The main result is that a Seifert integral homology sphere which is neither S^3 nor the Poincaré homology sphere, always admits a taut foliation. Nevertheless, the result is completely different when we only suppose that the rational homology of the manifold is trivial: whatever may be the number of exceptional fibers (greater than or equal to 3), there exist infinitely

many manifolds without taut foliations, and there exist infinitely many manifolds which admit one. Moreover, we will discuss the relations between the geometry and the existence of taut foliation.

Charalampos Charitos (Athens), *Generalized Teichmüller space of non-compact* 3*-manifolds and Mostow rigidity.*

Consider a 3-dimensional manifold N obtained by gluing a finite number of ideal hyperbolic tetrahedra via isometries along their faces. By varying the isometry type of each tetrahedron but keeping fixed the gluing pattern we define a space T of complete hyperbolic metrics on N with cone singularities along the edges of the tetrahedra. We prove that T is homeomorphic to a Euclidean space and we compute its dimension. By means of examples, we examine if the elements of T are uniquely determined by the angles around the edges of N. In particular, in the present talk we will present explicitly the space T when N is the complement of the Whitehead link.

Cameron Gordon (Austin), *Seifert fibered Dehn filling.*

It is conjectured that if M is a hyperbolic 3-manifold having non-hyperbolic Dehn fillings $M(\alpha)$ and $M(\beta)$ with $\Delta(\alpha, \beta) > 5$ then M is one of four specific manifolds. This is known if neither $M(\alpha)$ nor $M(\beta)$ is a small Seifert fibered space. In the case where $M(\alpha)$ is small Seifert and $M(\beta)$ is toroidal the conjecture implies that either $\Delta(\alpha, \beta) \leq 5$ or M is the figure eight knot exterior. As a step in the direction of proving this we show that if the essential punctured torus in M with boundary-slope β is not a fiber or semi-fiber for M and does not have exactly two boundary components then $\Delta(\alpha, \beta) \leq 5$. (This is joint work with Steve Boyer and Xingru Zhang.)

Marc Lackenby (Oxford), *Some elementary problems in knot theory.*

Some very simply-stated problems in knot theory remain unresolved. Can one find good upper and lower bounds on the number of Reidemeister move required to pass between two diagrams for a link? Is crossing number additive under connected sum? More generally, how is the crossing number of a satellite knot related to the crossing number of its constituent knots? Can

one determine whether a knot has unknotting number one? In my talk, I will provide some partial answers to these and related problems.

Feng Luo (Rutgers), *A dilogarithm identity on moduli space of surfaces.*

Given any closed hyperbolic surface of a fixed genus, we establish an identity involving dilogarithm of lengths of simple closed geodesics in all embedded pairs of pants and one-holed tori in the surface. One may consider this as a counter part of McShane's identity for closed hyperbolic surfaces. This is a joint work with Ser Peow Tan.

Serguei Nechaev (Orsay), *Topological correlations in dense lattice trivial knots.*

We have investigated the topological correlations in closed unknotted loop (polymer chain) in a compact state using Jones invariants. We have shown that any subpart of a trivial knot is weakly entangled. All sub-parts of the chain try not to spread over the volume and stay segregated on all scales resembling the Peano curve embedded in 3D space. This structure explains many important questions existing over the years, concerning functioning and packing the DNA in a chromosome. For example, it elucidates the question concerning for copying the genetic information: how to allow DNA (which could reach the length of 2 m) to fold and unfold extremely rapidly in regions of sizes less than the cell nucleus.

Giandomenico Palumbo (Pavia), *Post Quantum cryptography from mutant prime knots.*

By resorting to basic features of topological knot theory we propose a (classical) cryptographic protocol based on the 'difficulty' of decomposing complex knots generated as connected sums of prime knots and their mutants. The scheme combines an asymmetric public key protocol with symmetric private ones and is intrinsically secure against quantum eavesdropper attacks.

Ekaterina Pervova (Pisa), *Colored Turaev-Viro invariants for links in arbitrary 3-manifolds.*

We consider certain invariants of links in 3-manifolds, obtained by a specialization of the Turaev-Viro invariants of 3-manifolds, that we call colored Turaev-Viro invariants. We analyze some basic properties of these invariants, including the behavior under connected sums of pairs away and along links. These properties allow us to provide examples of links in the three-dimensional sphere having the same HOMFLY polynomial and the same Kauffman polynomial but distinct Turaev-Viro invariants, and similar examples for the Alexander polynomial. Finally, we establish a relation between the Turaev-Viro invariants of (M, L) and (the absolute value of) the Witten-Reshetikhin-Turaev invariants of (M, L'), where L' is L endowed with an arbitrary framing. (Joint with Carlo Petronio)

Jessica Purcell (Brigham Young), *The geometry of unknotting tunnels.*

An unknotting tunnel is an arc in a 3-manifold M with torus boundary, such that the complement of the tunnel in M is a handlebody. Classically, one can "unknot" a knot or link by pulling its diagram along an unknotting tunnel. In 1995, Adams, and Sakuma and Weeks, asked three questions concerning the geometry of unknotting tunnels in a hyperbolic 3-manifold: Are they geodesic? Do they have bounded length? Are they canonical? While the answer to the first question is still open, we will describe fairly complete answers to all three questions in the case where M is created by a "generic" Dehn filling. As an application, there is an explicit family of knots in the 3-sphere whose tunnels are arbitrarily long. This is joint with Daryl Cooper and David Futer.

Fionntan Roukema (Pisa), *Dehn surgery on the minimally twisted* 5*-chain link.*

We will present a description of the exceptional Dehn surgeries on the minimally twisted 5-chain link. We will see that, in some natural sense, there are only 21 such surgeries. The minimally twisted 4-chain link plays an integral role in the description of *all* exceptional surgeries on the minimally twisted 5-chain link and we will indicate how this can be utilized to describe exceptional pairs. This talk is based on joint work with Carlo Petronio and Bruno Martelli.

Nabil Sayari (Moncton), *Klein bottles and Dehn fillings.*

Let M be the exterior of a hyperbolic link $K \cup L$ in a homology 3-sphere Y, such that the linking number $\mathrm{lk}(K, L) \neq 0$. Denote by $M(r)$ the 3-manifold obtained by r-Dehn filling along K, and g the genus of the knot L. In this talk we show that if $M(r)$ contains a Klein bottle, then there is an upper bound on $\Delta(1/0, r)$ which depends on $\mathrm{lk}(K, L)$ and g.

Saul Schleimer (Warwick), *Census building using Twister and SnapPy.*

This is joint work with Tracy Hall and Mark Bell. We have written a computer program called Twister that builds three-manifolds based on Dehn twist and half-twist data. After reviewing previous censuses, I will briefly discuss the inner workings of Twister. Feeding the output of Twister into SnapPy one can rapidly produce censuses of hyperbolic manifolds: for example all genus 2 surfaces bundles over the circle with pseudo-Anosov monodromy of length at most 10. As another application, Mark Bell has found "canonical" monodromies for 40% of the fibered knots listed at KnotInfo.

Anastasiia Tsvietkova (Knoxville), *Investigating hyperbolic link complements.*

As a result of Thurston's Hyperbolization Theorem, many 3-manifolds have a hyperbolic metric or can be decomposed into pieces with hyperbolic metric (W. Thurston, 1978). In particular, Thurston demonstrated that every knot in S^3 is a torus knot, a satellite knot or a hyperbolic knot and these three categories are mutually exclusive. It also follows from work of Menasco that an alternating link represented by a prime diagram is either hyperbolic or a $(2, n)$-torus link.
A new method for computing the hyperbolic structure of the complement of a hyperbolic link, based on ideal polygons bounding the regions of a diagram of the link rather than decomposition of the complement into ideal tetrahedra, was suggested by M. Thistlethwaite. The talk will introduce the basics of the method. Some applications will be discussed, including a surprising rigidity property of certain tangles, a new numerical invariant for tangles, a number field that is an invariant of a hyperbolic link, and an algorithm for computing hyperbolic volume.

Vladimir Vershinin (Montpellier), *On Vassiliev invariants of braid groups of the sphere.*

We construct a universal Vassiliev invariant for braids of the sphere and the mapping class groups of the sphere with n punctures. The case of a sphere is different from the classical braid groups or braids of oriented surfaces of genus strictly greater than zero, in the case of a sphere Vassiliev invariants in a group without 2-torsion does not distinguish elements of braids.