

## Scientific Report on the Semester

### Stochastic Analysis, Stochastic Partial Differential Equations and Applications Centro di Ricerca Matematica E. De Giorgi March - July 2006

Stochastic Analysis is an important field, developed recently and growing rapidly, which is based on a deep mixing of functional analytic tools (infinite-dimensional analysis) with Probability. Its increasing importance is due to the numerous applications in apparently very different fields as: biology, chemistry, finance, mathematics, physics.

Topics discussed in the semester include: random dynamical systems, stochastic partial differential equations, Dirichlet forms, Malliavin calculus, systems of interacting particles and others.

The scope of the semester was to discuss the “state of the art” of all these topics by emphasizing the numerous connections among them.

We started with Malliavin calculus and random dynamical systems. Then we continued with stochastic partial differential equations. We paid much attention to interacting particles evolving according to irreversible phenomena in non equilibrium statistical mechanics.

Finally, we devoted the last part of the semester to equations of hydrodynamics such as Navier-Stokes and Burgers equations perturbed by noise and turbulence, which is an actual leading subject in statistical hydrodynamics.

The activities of the semester were devoted in particular to the following topics.

- (i) *Stochastic analysis in infinite dimensions.*
- (ii) *Stochastic partial differential equations.*
- (iii) *Dirichlet forms and Kolmogorov equations.*
- (iv) *Particle systems and statistical mechanics.*
- (v) *Stochastic fluid dynamics.*

They included

- (1) **7 courses** held by P. Malliavin (Paris VI), J. Zabczyk (Polish Academy of Sciences), B. Goldys (University of South Wales), C. Landim (IMPA, Brazil), A. Debussche (École Normale Supérieure, Antenne de Bretagne), F. Guerra (Università di Roma “la Sapienza”), E. Bolthausen (Universität Zürich) and the “Galileo Lectures” delivered by Y. G. Sinai.

(2) **8 shorter series of lectures** delivered by M. Freidlin (University of Maryland), T. Funaki (University of Tokyo), A. Shirikyan (Université de Paris Sud), S. Kuksin (University of Edinburgh), F. Martinelli (Università di Roma 3), N. V. Krylov (University of Minnesota), J. Mattingly (Duke University), P. E. Souganidis (University of Texas).

(3) **4 workshops** on:

- Random Dynamical Systems in Infinite Dimensions, March 28-29, 2006. Local Committee: G. Da Prato (SNS Pisa), F. Flandoli (Università di Pisa).
- Stochastic Partial Differential Equations, April 3-8, 2006. Local Committee: Sandra Cerrai (Università di Firenze), Lorenzo Zambotti (Politecnico di Milano), Luciano Tubaro (Università di Trento)
- Hydrodynamic limits and Particle Systems, June 5-10, 2006. Local Committee: Lorenzo Bertini (Università di Roma “La Sapienza”), Marco Isopi (Università di Roma “La Sapienza”), Lorenzo Zambotti (Politecnico di Milano).
- Stochastic Fluid Mechanics and SPDEs, July 24-28, 2006. Local Committee: Massimiliano Gubinelli (Università di Pisa), Marco Romito (Università di Firenze).

The **Scientific committee** was formed by  
Giuseppe Da Prato Scuola Normale Superiore,  
Franco Flandoli Università di Pisa,  
Giovanni Jona-Lasinio Università di Roma “La Sapienza”,  
Etienne Pardoux Université de Provence,  
Michael Röckner Faculty of Mathematics, Bielefeld University

There were many local and foreign participants from numerous countries including several young participants which got a grant from the De Giorgi Center. A list is given at the end of this report.

Let us illustrate all these activities by a brief description of the content of the courses as well as of the titles of talks.

## Courses

- **P. Malliavin**, Stochastic Analysis in Infinite Dimension.

*Abstract.* Gaussian Probability spaces. Gross-Stroock Sobolev spaces, divergences. Ordinary differential equations on Gaussian probability spaces. Ornstein-Uhlenbeck process. Smoothness of law for non degenerated random variables. Stochastic Calculus of Variations for a finite dimensional SDE. The group of automorphisms of a filtered Gaussian Probability Space. Backward and forward regularity for finite dimensional elliptic heat semi-group. Contracting semi-group and invariant measure. Virasoro algebra, diffeomorphism group of the circle, Sato Grassmannian.

Brownian motion on the group of diffeomorphisms of the circle, its modulus of continuity. Differential geometry on the group of homeomorphisms of the circle. Quasi-invariance of heat measure on the diffeomorphisms group of the circle. Discrete series representation of Virasoro algebra : unitarizing measures.

- **J. Zabczyk**, Markov processes and stochastic equations.

*Abstract.* Markov processes in discrete time. Markovian semigroups and Courge theorem. Levy processes. Properties of trajectories. Stochastic equations in finite and infinite dimensions.

- **B. Goldys**, Elements of Ergodic Theory for SPDE's.

*Abstract.* Review of main results on Ergodic Theory. Markov semigroups associated to SPDE's and their generators in spaces of continuous functions endowed with the mixed topology. Approximation theorems like Trotter-Kato and Trotter-Lie theorems and their applications to the construction of generators of strongly continuous (in an appropriate sense) semigroups by analytic methods.

- **C. Landim**, Central limit theorems for Markov processes.

*Abstract.* Central limit theorem for Markov chains. Equilibrium central limit theorem for a tagged particle in interacting particle systems: the reversible case, the mean zero case and the asymmetric case in  $d \geq 3$ . Smoothness of the diffusion coefficient: the generalized duality. Non equilibrium central limit theorem for the symmetric simple exclusion process in  $d = 1$ .

- **F. Guerra**, Introduction to the theory of Spin glasses.

*Abstract.* Statistical ensembles. Examples of statistical mechanics systems at equilibrium. The Curie-Weiss model for the ferromagnetism. An example of disordered system: the REM model with random energies. The Derrida-Ruelle cascade as a system of statistical mechanics. The Sherrington-Kirkpatrick spin glass model. The Parisi representation for the free energy and the states. General methods of interpolation. The broken replica symmetry bounds. The Aizenman-Sims-Starr variational principle. The overlap locking phenomenon. The Talagrand proof of the Parisi representation for the free energy. Probabilistic aspects of the Parisi theory. Analysis of the high temperature regime. The phenomenon of the spontaneous breaking of the replica symmetry. Overlap fluctuations. Considerations on short range spin glasses.

- **A. Debussche**, Introduction to dispersive SPEs.

*Abstract.* The Strichartz estimates for stochastic dispersive PDEs including Nonlinear Schrodinger Equations (NLS). Stochastic NLS equation with an additive and multiplicative noise: basic well-posedness results, qualitative influence of a noise on blow-up phenomena in the case of a critical

or supercritical nonlinearity, the strong effect of spatially smooth noise. Large deviation estimates and estimate of the probability errors in soliton transmission arising in optical fibers. Numerical simulations and numerical analysis of the stochastic NLS equations. Their connection with infinite dimensional Kolmogorov equation. Korteweg-de Vries equation and the associated solitons when a noise perturbs the dynamics.

- **E. Bolthausen**, Random walk in random environment.

*Abstract.* A new approach to investigate random walks in random environments in dimension 3 and larger. Detailed analysis of the exit distributions. Some results on the existence of invariant measures for random walks in random environments, adapting a method originally found to prove diffusivity of directed polymers.

- **Y. G. Sinai**, Selected problems from Fluid Dynamics.

*Abstract.* Three dimensional Navier-Stokes system and problems related to the appearance of blow ups of solutions. Two dimensional Navier-Stokes system, Burgers system and statistics of shock waves.

### Short series of lectures

- **M. Freidlin**. Averaging principle and long time behavior of weakly coupled oscillators.
- **T. Funaki**. Some topics on an effective interface model.
- **N. Krylov**. On the maximum principle for SPDEs, the continuity of their solutions, and the law of square root for Brownian motion.
- **P. Souganidis**. Stochastic Homogenization for nonlinear first and second-order PDEs in stationary ergodic media.
- **A. Shirikyan**. Controllability of 3D Navier-Stokes equations.
- **S. B. Kuksin**. Khasminski-Whitham averaging for randomly perturbed KdV equation.
- **F. Martinelli**. Some topics in Glauber dynamics for spin systems.
- **J. C. Mattingly**. Asymptotic strong Feller and Ergodicity of SPDEs.

### Workshops Talks

- 1 *Workshop on “Random Dynamical Systems in Infinite Dimensions”.*
  - P. Malliavin, Regularity of the heat semigroup on the diffeomorphism group of the circle
  - Y. Le Jan, Between coalescence and diffusion
  - Z. Brzezniak, Asymptotic compactness for 2D Stochastic Navier-Stokes equations with multiplicative noise
  - P. Kloeden, Discretization of asymptotically stable stationary solutions of delay.
  - H. Crauel, Stabilization by rotation
  - M. Gordina, Infinite dimensional stochastic differential equations of Ornstein-Uhlenbeck type
  - E. Hausenblas, SPDEs driven by Poisson Random measure
  - D. Elworthy, Decomposition of diffusions
  - J. Robinson, Embedding theorems for random attractors
  - B. Schmalfuss, Invariant manifolds for stochastic wave equation
  - T. Caraballo, Synchronization of a stochastic reaction-diffusion system on a thin two-layer domain
  - J. Langa, Flattening, squeezing and the existence of attractors for random dynamical systems
  - O. Tearne, Collapse of Random Attractors for Gradient SDE
- 2 *Workshop on “Stochastic Partial Differential Equations”.*
  - P. Malliavin, Low regularity Stochastic flows and infinite dimensional Lie algebras
  - V. Barbu, Stochastic wave equation with non-linear damping
  - S. Tindel, Towards a pathwise definition of stochastic PDEs
  - B. Goldys, Generation of diffusion semigroups in spaces of continuous functions
  - G. Tessitore, On a class of stochastic optimal control problems related to BSDEs with quadratic growth
  - E. Priola, On bounded solutions to convolution equations
  - E. Gautier, Escape from a basin of attraction for weakly damped stochastic non-linear Schrödinger equations perturbed by small noise
  - C. Marinelli, Optimal control of stochastic differential equations with memory
  - C. Mueller, The speed of a random travelling wave
  - A. Stuart, Stochastic PDEs for sampling conditioned diffusions I
  - R. Buckdhan, Stochastic Taylor expansion and stochastic viscosity solutions for non-linear SPDEs

B. Maslowski, Ornstein-Uhlenbeck bridge and applications to semilinear SPDEs

M. Fuhrman, Some results on backward stochastic differential equations in infinite dimensions

T.S. Zhang, Large deviations for SPDEs driven by Levy processes

J. Voss, Stochastic PDEs for sampling conditioned diffusions II

H. Lisei, SPDEs driven by fractional Brownian motion

Leonid Mytnik, On uniqueness for stochastic heat equations with non-Lipschitz coefficients

P. Souganidis, Stochastic homogenization

R. Dalang, Hitting probabilities for the non-linear stochastic heat equation

F. Russo, Some parabolic PDEs whose drift is an irregular random noise in space

W. Stannat, Stability of the optimal filter w.r.t. its initial condition

L. Zambotti, Stochastic PDEs with reflection and conservation of the space average

M. Gordina, Infinite dimensional stochastic differential equations of Ornstein-Uhlenbeck type

B. Rdiger, Stochastic differential equations with non Gaussian additive noise on infinite dimensional spaces

A. Rusinek, Invariant measures for a class of stochastic differential equations

M. Sanz-Solé, Approximations of elliptic SPDEs in high dimensions

R. Leandre, Malliavin calculus of Bismut type without probability and its applications

R. Sowers, Random ducks

A. De Bouard, Soliton dynamics for stochastic perturbations of long wave equations

F. Gozzi, Verification theorems for infinite dimensional stochastic optimal control problems

X.M. Li, Some remarks on perturbation of stochastic integrable Hamiltonian systems

S. Bonaccorsi, Stochastic evolution equations with dynamical boundary conditions

E. Hausenblas, The numerical approximation of the reflected stochastic heat equation

L. Accardi, Connections within higher powers white noise and enlarged Virasoro algebra

- A. Millet, On the speed of convergence of discretization schemes for evolution equations
- M. Hairer, A spectral gap for the stochastic Navier-Stokes equations
- A. Lunardi, Ornstein-Uhlenbeck operators with periodic coefficients
- P. Vuillermot, Some recent results regarding fractional stochastic partial differential equations
- M. Dozzi, Blow-up behaviour of certain PDE's and SPDE's of parabolic and hyperbolic type
- D. Bloemker, On a Model from Surface-Growth
- S. Cerrai, On the ergodic properties of some classes of SPDEs
- 3 *Workshop on "Hydrodynamic limits and Particle Systems"*.
- Mark Freidlin, Reaction-advection in incompressible fluid
- Claudio Landim, Nonequilibrium current large deviations in interacting particle systems
- Giambattista Giacomin, Brownian scaling and universality for directed polymers
- Alessandra Faggionato, Mott variable-range random walk
- Tobias Kuna, A Bochner type theorem for Bogoliubov functionals
- Massimiliano Gubinelli, Gibbs measures on Brownian paths
- Patricia Gonçalves, Central limit theorem for a tagged particle in asymmetric simple exclusion.
- Milton Jara, Hydrodynamic limit for diffusive systems with bond disorder
- Boguslaw Zegarlinski, Linear and Nonlinear Problems in Large interacting Systems
- Stefano Olla, Microscopic models for heat conduction
- Yuri Kondratiev, Scaling limits for Kawasaki dynamics in continuum
- Cedric Bernardin, Large deviations for a heat conduction model
- Lorenzo Zambotti, Fluctuations for  $\nabla\phi$  interface models near a wall and with conservation of the space average
- Francesco Caravenna, A renewal theory approach to weakly inhomogeneous polymer models
- Zeev Sobol, Method of Lyapunov functions with application to QG equation
- Oleksandr Kutovyy, On non-equilibrium stochastic dynamics for interacting particle systems in continuum
- Francesco Guerra, Overview about rigorous methods and results in mean field spin glass theory

Thomas Mountford, A convergence result for one dimensional exclusion processes

Alberto Gandolfi, Statistical mechanics and transmission on trees

Jean-Dominique Deuschel, Scaling limits of non nearest neighbor pinning models in  $(1+1)$ -dimension

Lorenzo Bertini, Large deviations for weakly asymmetric stochastic lattice gases

Fabio Martinelli, Kinetically constrained spin models: rigorous results

Sandra Cerrai, On the Smolukowski- Kramers approximation for a general class of SPDE's

Marco Isopi, Scaling limits of 1-d dynamics at low temperature

Martin Grothaus, Strong Feller properties and applications to continuous N-particle systems with reflecting boundary condition

Gustavo Posta, Ecological Equilibrium for an Unbounded Contact Process

Seung Lee, Reflected Rouse Model

Tadahisa Funaki, The Brascamp-Lieb inequality and its applications

Giovanni Jona-Lasinio, An informal introduction to molecular motors

Michael Röckner, Stochastic porous media and fast diffusion equations

#### 4 *Workshop on "Stochastic Fluid Mechanics and SPDEs"*

Ana Bela Cruzeiro, Brownian motion on the diffeomorphisms group and global solution of a 2D stochastic Euler equation.

Jonathan C Mattingly, Ergodicity and a Spectral gap for the 2D Navier Stokes equations.

Zdzislaw Brzezniak, Stochastic Burgers equations driven by fractional Laplacian and space-time white noise.

Laura Maria Morato, Stochastic model of a quantum fluid and generation of vortex lines.

Zeev Sobol, Stochastic equations in fluid dynamics.

Hannelore Inge Lisei, Navier - Stokes Equations Perturbed by Fractional Brownian Motion.

Archil Gulisashvili, Stochastic volatility models: Asymptotic behavior of distribution densities.

Viorel Barbu, Local Ergodic Behaviour of a Stabilized 2-D Navier-Stokes Equations Perturbed by Noise.

Etienne Pardoux, Homogenization of possibly degenerate periodic PDEs; a probabilistic approach.

Giovanni Jona-Lasinio, Violation of time reversal invariance and long range space correlations.



Sergio Albeverio, Some new developments in infinite-dimensional analysis.

Arnaud Debussche, Markov solutions for the 3D stochastic Navier-Stokes equations with state dependent noise.

Michael Röckner, Elliptic and parabolic PDEs for measures.

Michael Scheutzow, Dispersion of sets under a stochastic flow.

Szymon Peszat, Passive tracer dynamics.

Armen Shirikyan, Qualitative properties of 3D Navier-Stokes dynamics.

Feng-Yu Wang, Harnack Inequality and Applications for Stochastic Generalized Porous Media Equations.

Peter Imkeller, Transitions between meta-stable states induced by Levy noise.

Jinqiao Duan, Effective Macroscopic Dynamics of Stochastic Partial Differential.

Peter Kotelenez, Macroscopic Limit for the Vorticity Distribution in a 2D Fluid.

Martin Hairer, Intermittent behaviour of the stochastic Burgers equations.

Mauro Mariani, Large deviations principle for stochastically perturbed conservation laws.

Vahagn Nersesyan, Polynomial mixing for the complex Ginzburg-Landau equation perturbed by a random force at random times.

Aubrey Truman, The Maxwell set and other singularities of Stochastic Burgers equations.

Padma Sundar, Large Deviations for Stochastic Navier-Stokes equations.

Dirk Blömker, Stochastic Dynamics near a Bifurcation - Amplitude Equations.

Marco Romito, Markov solutions and their properties for the stochastic 3D Navier-Stokes equations.

Benedetta Ferrario, On a stochastic version of Prouse model in fluid dynamics.

Cyril Odasso, Spectral gap and asymptotic strong Feller property for Reaction Diffusion equation driven by a Levy-type noise.

## List of participants including junior grants

Kasra Alishahi, Sharif university of technology July 2006

Alessio Ansuini, Università di Roma 1 5-9 June 2006

Giada Basile, Università di Firenze 5-9 June 2006

Arnaud Basson, École normale supérieure (Paris) 23-29 July 2006

Cedric Bernardin, École Normale Supérieure, Lyon 10 May-10 June 2006

Lorenzo Bertini, Università di Roma La Sapienza July 2006  
 Hakima Bessaih, University of Wyoming, 6-27 June 2006  
 Alessandra Bianchi, Università di Roma 3 5-9 June 2006  
 Fabrice Blache, University of Bonn 2-12 April 2006  
 Pavel Bubak, University of Warwick 23-28 July 2006  
 Michael Caruana, University of Oxford 11 June 2006 - 30 July 2006  
 Roy Cerqueti, Università “La Sapienza” - Roma  
 Fulvia Confortola, Politecnico di Milano 3 -7 April 2006  
 Daniel Conus, EPFL-Lausanne 3-7 April 2006  
 Alessandra Faggionato, Università di Roma 1. 10 May-10 June 2006  
 Davide Gabrielli, Università de L’Aquila 10 May 2006 - 10 June 2006  
 Eric Gautier, Ecole Normale Supérieure Cachan 1-11 April and  
 30 May- 17 June 2006  
 Ana Patricia Goncalves, IMPA 22 May-10 June 2005  
 Maria Gordina, University of Connecticut  
 25 March -7 April and 20-29 July 2006  
 Ludovic Goudenège, École Normale Supérieure de Cachan  
 10 July-03 August 2006  
 Giuseppina Gattereri, Politecnico di Milano 3-7 April 2006  
 Massimiliano Gubinelli, Università di Pisa 1 March-31 July 2006  
 Roberto Guenzani, Università di Milano 6-10 June 2006  
 Markus Haase, University of Leeds 2-8 April 2006  
 Erika Hausenblas, University of Salzburg 2-15 April and 16-27 July 2006  
 Milton Jara, IMPA 10 May-10 June 2006  
 Nicola Kistler, University of Zurich 10-14 July 2006  
 Tobias Kuna, University of Bielefeld, 16-19 May 2006 and 4-9 June 2006  
 Oleksandr Kutovyy, University of Bielefeld 10 May-11 June 2006  
 Seung Lee, Ohio State University 27 March-10 June 2006  
 Luigi Manca, Scuola Normale Superiore di Pisa  
 Maria Elvira Mancino, DIMAD University of Firenze  
 Iuliana Marchis, Babes-Bolyai University 2-9 April 2006  
 Mauro Mariani, Università degli studi di Roma 3-29 July 2006  
 Carlo Marinelli, University of Bonn 3 April-2 May 2006  
 Gabriela Marinoschi, University of Bucharest 24-28 July 2006  
 Federica Masiero, Politecnico di Milano  
 Ida Minelli, Università degli Studi di Padova 15-19 May 2006  
 Philippe Mounaix, CNRS, Ecole Polytechnique 5-10 June 2006  
 Vahagn Nersesyan, Université de Paris-Sud XI, France 1 -28 July 2006  
 Cyril Odasso, Ecole Normale Supérieure de Cachan 24-28 July 2006  
 Grigorios Pavliotis, Imperial College London 2-9 April 2006  
 Marco Ribezzi, Università di Roma 1 5-9 June 2006  
 Anna Rusinek, Institute of mathematics of the Polish Academy of Sciences  
 2 - 13 April 2006  
 Elena Sartori, Università di Padova 5-9 June 2006  
 Oliver Tearne, University of Warwick. 27-31 March 2006  
 Marco Tolotti, Università Bocconi, Milano 5 -10 June 2006

Gerald Trutnau, Universität Bielefeld 22 May-22 June 2006  
Ciprian Tudor, Université de Paris 1 16 June-15 July 2006  
Jochen Voss, University of Warwick 1-30 April 2006  
Giacomo Ziglio, Università di Trento 2 -8 April 2006 and  
30 April-03 June 2006