Terzo Incontro Italiano di Teoria dei Numeri

Centro di Ricerca Matematica Ennio De Giorgi

Pisa, 21-24 Settembre 2015

Relazione Scientifica

Lista dei partecipanti

Abstracts

Contents

Relazione riassuntiva

Comitato Scientifico:

Prof. Massimo Bertolini	*Universität Duisburg-Essen*
Prof. Roberto Dvornicich	*Università di Pisa*
Prof. Alberto Perelli	*Università di Genova*
Prof. Carlo Viola	*Università di Pisa*
Prof. Umberto Zannier	*Scuola Normale Superiore di Pisa*

Comitato Organizzatore:

Prof. Ilaria Del Corso	*Università di Pisa*
Dr. Andrea Bandini	*Università di Parma*

La conferenza è stata ospitata dal Centro di Ricerca Matematica Ennio De Giorgi e ha avuto i seguenti sponsor: CRM De Giorgi, GNSAGA e Università di Pisa. Grazie al loro contributo abbiamo potuto pagare le spese di alloggio a 30 conferenzieri (di cui 17 tra i "giovani" degli short talks) e a 5 giovani dottorandi, alcuni viaggi dei conferenzieri, i pasti presso la mensa della Scuola Normale Superiore di Pisa per tutti i partecipanti, una cena sociale e tutte le altre spese necessarie al buon funzionamento di tutti gli aspetti della conferenza.

Partecipanti. La conferenza ha avuto una partecipazione molto ampia, anche oltre le aspettative (63 persone, 16 provenienti dall'estero) grazie all'adesione della maggior parte dei docenti di teoria dei numeri italiani e alla partecipazione di moltissimi giovani dottorandi e post-doc italiani, molti dei quali sono attualmente all'estero. Alleghiamo la lista dei partecipanti

Programma. Ci sono state 14 conferenze generali e 26 conferenze brevi, tenute in due sessioni parallele. Gli abstract di tutte le conferenze sono allegati. Sia le conferenze lunghe che quelle brevi hanno presentato risultati significativi in aree di ricerca attuali e di grande prospettiva. Soprattutto nelle conferenze lunghe gli oratori si sono impegnati per rendere accessibile ad una platea di vasti e diversi interessi il quadro generale delle loro ricerche e dei possibili sviluppi futuri, permettendo così ai più giovani di avere una panoramica di tutta la teoria dei numeri italiana e di stabilire nuovi contatti con gli esperti del settore.

Grazie a questa ampia partecipazione ed all'impegno di tutti i conferenzieri, il congresso è stato un importante momento di incontro e di scambio scientifico, che pensiamo possa dare origine anche a nuove collaborazioni nel prossimo futuro.

Per rendere accessibili ad un ancora più vasto pubblico i risultati del congresso, è prevista la pubblicazione di un volume dedicato agli Atti, da parte della Rivista di Matematica della Università di Parma (volume speciale del 2016).

Gli Organizzatori

Ilaria Del Corso

Andrea Bandini

PROGRAMMA IITDN2015

	9.30-10.20	**10.50-11.40**	**11.50-12.40**	**14.30-14.55**	**15.00-15.25**	**15.30-15.55**	**16.30-16.55**	**17.00-17.25**
Lunedì 21	Baldassarri	Chiarellotto	Pappalardi	Viada (14.30-15.20)		Cobbe Murru	Veneziano Maffucci	Paladino Coppola
Martedì 22	Andreatta	Longo	Vigni	Barroero Bazzanella	Capuano Melfi	Mantova Monopoli	Centeleghe Righetti	Caldarola Molteni
Mercoledì 23	Iovita	Languasco	Zaccagnini	Masoero Talamanca	Valentino Dose	Venerucci Demangos	Rosso Turchet	Seveso Meleleo
Giovedì 24	Amoroso	Gasbarri	Corvaja	Pellarin (14.30-15.20)				

Registrazione: Lunedì 8.15-9.30 in Aula Dini (Palazzo del Castelletto)
Tutti i giorni: 10.20-10.50 Pausa Caffè
12.40-14.30 Pranzo
16.00-16.30 Pausa Caffè (escluso Giovedì)

AULE: Main Speakers in Aula Dini (Palazzo del Castelletto)
Short Talks in Aula Russo e Aula Contini (Palazzo Carovana)

Lista dei partecipanti

1.	**Francesco Amoroso**	*Université de Caen Basse-Normandie*
2.	**Fabrizio Andreatta**	*Università Statale di Milano*
3.	**Francesco Baldassarri**	*Università di Padova*
4.	**Andrea Bandini**	*Università di Parma*
5.	**Fabrizio Barroero**	*Scuola Normale Superiore di Pisa*
6.	**Danilo Bazzanella**	*Politecnico di Torino*
7.	**Mattia Cafferata**	*Università di Parma*
8.	**Fabio Caldarola**	*Università della Calabria*
9.	**Leonardo Cangelmi**	*Università di Chieti-Pescara*
10.	**Marco Cantarini**	*Univeristà di Parma*
11.	**Laura Capuano**	*Scuola Normale Superiore di Pisa*
12.	**Tommaso Centeleghe**	*Universität Heidelberg*
13.	**Sara Checcoli**	*Institut Fourier, Grenoble*
14.	**Bruno Chiarellotto**	*Università di Padova*
15.	**Andrea Ciappi**	*Scuola Normale Superiore di Pisa*
16.	**Alessandro Cobbe**	*Universität der Bundeswehr München*
17.	**Giovanni Coppola**	*Università di Salerno*
18.	**Antonio Corbo Esposito**	*Università di Cassino e del Lazio meridionale*
19.	**Pietro Corvaja**	*Università di Udine*
20.	**Edoardo Coscelli**	*Università degli Studi di Milano*
21.	**Giuliana Davidoff**	*Mount Holyoke College*
22.	**Ilaria Del Corso**	*Università di Pisa*
23.	**Luca Demangos**	*UNAM, Mexico*
24.	**Valerio Dose**	*Università di Roma "Tor Vergata"*
25.	**Roberto Dvornicich**	*Università di Pisa*
26.	**Alessandro Gambini**	*Università di Ferrara*
27.	**Carlo Gasbarri**	*Université de Strasbourg*
28.	**Adrian Iovita**	*Università di Padova*
29.	**Alessandro Languasco**	*Università di Padova*
30.	**Antonino Leonardis**	*Scuola Normale Superiore di Pisa*
31.	**Matteo Longo**	*Università di Padova*
32.	**Riccardo Walter Maffucci**	*King's College London*
33.	**Francesca Malagoli**	*Università di Pisa*
34.	**Vincenzo Mantova**	*Università di Pisa*
35.	**Raffaele Marcovecchio**	*Università di Chieti-Pescara*
36.	**Daniele Masoero**	*Università degli Studi di Genova*
37.	**Stefano Mastrostefano**	*Università di Cassino e del Lazio Meridionale*
38.	**Giulio Meleleo**	*Università di Roma 3*
39.	**Giuseppe Melfi**	*University of Applied Sciences of Western Switzerland*
40.	**Olaf Merkert**	*Scuola Normale Superiore di Pisa*

41. **Giuseppe Molteni** *Università Statale di Milano*
42. **Francesco Monopoli** *Università Statale di Milano*
43. **Marina Monsurrò** *Università Europea di Roma*
44. **Nadir Murru** *Università di Torino*
45. **Frances Odumodu** *Université Bordeaux and Università di Padova*
46. **Laura Paladino** *Università di Pisa*
47. **Francesco Pappalardi** *Università di Roma 3*
48. **Maria Rosaria Pati** *Università di Pisa*
49. **Federico Pellarin** *Université Jean Monnet, Saint-Etienne*
50. **Alberto Perelli** *Università di Genova*
51. **Francesco Pinna** *Università di Firenze*
52. **Mattia Righetti** *Università di Genova*
53. **Giovanni Rosso** *FWO KU Leuven*
54. **Marco Seveso** *Università Statale di Milano*
55. **Valerio Talamanca** *Università di Roma 3*
56. **Amos Turchet** *Chalmers University of Technology and Göteborg University*
57. **Maria Valentino** *Scuola Normale Superiore di Pisa*
58. **Rodolfo Venerucci** *Universität Duisburg-Essen*
59. **Francesco Veneziano** *Universität Basel*
60. **Evelina Viada** *ETH Zurich*
61. **Stefano Vigni** *Università di Genova*
62. **Alessandro Zaccagnini** *Università di Parma*
63. **Umberto Zannier** *Scuola Normale Superiore di Pisa*

MAIN SPEAKERS

Risultati di altezza limitata per intersezioni di sottovarietà algebriche di un toro con famiglie di sottogruppi finitamente generati

Francesco Amoroso

Institution: *Université de Caen Basse-Normandie*

ABSTRACT: In questo seminario presenteremo recenti risultati di altezza limitata, ottenuti in collaborazione con D. Masser e U. Zannier, sull'intersezione con sottovarietà algebriche di un toro di famiglie a un parametro di sottogruppi finitemente generati.

Colmez conjecture on average

Fabrizio Andreatta

Institution: *Università Statale di Milano*

ABSTRACT: Colmez conjectured an explicit formula for the height of CM abelian varieties in terms of special values of L-functions. Recently Jacob Tsimerman showed that a weak form of Colmez conjecture would imply André-Oort's conjecture without the GRH. I will present recent work of myself, Eyal Goren, Ben Howard and Keerthi Madapusi Pera providing a proof of this weak form of Colmez conjecture.

A p-adically entire function with integral values on $\mathbb{Q}_p$ and additive characters of perfectoid fields

Francesco Baldassarri

Institution: *Università di Padova*

ABSTRACT: We give an essentially self-contained proof of the fact that a certain p-adic power series

$$\Psi = \Psi_p(T) \in T + Tp - 1\mathbb{Z}[Tp - 1] \, ,$$

which trivializes the addition law of the formal group of Witt p-covectors is p-adically entire and assumes values in $\mathbb{Z}_p$ all over $\mathbb{Q}_p$. We also carefully examine its valuation and Newton polygons. For any perfectoid field extension (K, p) of $(\mathbb{Q}_p, p)$ contained in $(\mathbb{C}_p, p)$, and any pseudo-uniformizer $\varpi = (\varpi(i))$ $i \geqslant 0$ of K, we consider the element

$$\pi = \pi(\varpi) := \sum_{i \geqslant 0} \varpi(i)\pi + \sum_{i<0} (\varpi(0))p - i\pi \in K \, .$$

We use the isomorphism between the Witt and the Cartier (hyperexponential) group over $\mathbb{Z}(p)$, which we extend to their p-divisible closures, and the properties of Ψ_p, to show that

the map $x \mapsto \exp \pi x$, a priori only defined for $v_p(x) > \frac{1}{p-1} - v_p(\pi)$, extends to a continuous additive character

$$\Psi\varpi : \mathbb{Q}_p \to 1 + K_{\circ\circ} .$$

A similar character for the cyclotomic p-extension of $\mathbb{Q}_p$ appears in Colmez' work. I will also give the numerical computation of the first coefficients of Ψ_p, for small p.

Monodromy criterium for good reduction of surfaces

Bruno Chiarellotto

Institution: *Università di Padova*

ABSTRACT: We want to address the problem of giving a criterion of good reduction for a semistable scheme defined over a complete DVR with mixed characteristic. The method we would like to use is the monodromy action. The monodromy will be used to detect good reduction but also the type of degeneration one can have when it is not smooth. We will focus on the case of K3 surfaces and we will give some hints for possible generalizations (joint work with Genaro H. Mada).

Stime del massimo comun divisore in aritmetica e in geometria

Pietro Corvaja

Institution: *Università di Udine*

ABSTRACT: In un lavoro con Y. Bugeaud e U. Zannier si dimostrò il seguente Teorema: dati due interi moltiplicativamente indipendenti $a > b > 1$ e un numero $\varepsilon > 0$, per ogni esponente n sufficientemente alto vale la maggiorazione $gcd(a^n - 1, b^n - 1) < exp(\varepsilon n)$. Si mostreranno generalizzazioni e applicazioni di queste stime, e disuguaglianze analoghe in geometria algebrica e geometria analitica complessa.

Differences between number fields and function fields

Carlo Gasbarri

Institution: *Université de Strasbourg*

ABSTRACT: It is well known that there should be an analogy between the arithmetic of rational points on varieties over global function fields and over number fields. This analogy should be confirmed also by some deep conjectures, like for instance Vojta conjectures. Nevertheless there are some substantial differences. I would like to make some comments on some of these differences. It is my opinion that once we better understand the reason of these differences, we will be able to push forward the analogies between the theories and the involved methodologies.

Modular sheaves and the geometry of the eigencurve

Adrian Iovita

Institution: *Università di Padova*

ABSTRACT: Together with F. Andreatta, V. Pilloni and G. Stevens we have constructed, attached to p-adic weights, modular sheaves whose global sections are the overconvergent modular forms defined by Coleman back in the 1996. Recently we have refined these constructions and proved a conjecture of Coleman regarding the geometry of the eigencurve over the compactified weight space.

On some exponential sums over prime powers and applications

Alessandro Languasco

Institution: *Università di Padova*

e-mail: languasc@math.unipd.it

2010 MSC: 11P32, 11P55, 11P05, 44A10, 33C10

ABSTRACT: Let Λ be the von Mangoldt function and N, $\ell > 1$ be integers. We will see some recent theorems on additive problems with prime and prime powers obtained by Languasco and Zaccagnini using the original Hardy-Littlewood circle method function, i.e.

$$\widetilde{S}_\ell(\alpha) = \sum_{n=1}^{\infty} \Lambda(n) e^{-n^\ell/N} e(n^\ell \alpha) \tag{1}$$

where $e(x) = \exp(2\pi i x)$. We'll compare such results with the weaker ones that can be obtained using finite sums like

$$S_\ell(\alpha) = \sum_{n=1}^{N} \Lambda(n) e(n^\ell \alpha) \tag{2}$$

this way explaining in which cases the (1)-approach could lead to sharper results.

References

[1] A. Languasco - A. Zaccagnini, *The number of Goldbach representations of an integer*, Proc. Amer. Math. Soc. **140** (2012), 795–804.

[2] A. Languasco - A. Zaccagnini, *A Cesàro Average of Hardy-Littlewood numbers*, J. Math. Anal. Appl. **401** (2013), 568–577.

[3] A. Languasco - A. Zaccagnini, *A Cesàro Average of Goldbach numbers*, Forum Mathematicum **27** (2015), 1945–1960.

[4] A. Languasco - A. Zaccagnini, *Sum of one prime and two squares of primes in short intervals*, Preprint (2014). Submitted. http://arxiv.org/abs/1406.3441.

[5] A. Languasco - A. Zaccagnini, *Short intervals asymptotic formulae for binary problems with primes and powers, I: density 3/2*, Preprint (2015). Submitted. http://arxiv.org/abs/1504.02271.

[6] A. Languasco - A. Zaccagnini, *Short intervals asymptotic formulae for binary problems with primes and powers, II: density 1*, Preprint (2015). Submitted. http://arxiv.org/abs/1504.04709

Variation of anticyclotomic Iwasawa invariants in Hida families

Matteo Longo

Institution: *Università di Padova*

ABSTRACT: Given a modular form f and a quadratic imaginary field K, one can form a p-adic L-function interpolating central critical values of the complex L-function of f twisted by characters of the anticyclotomic $\mathbb{Z}_p$-extension of K. Under suitable parity conditions, one shows that this p-adic L-function is non-zero, and we can consider its μ and λ Iwasawa invariants. When f lives in a Hida family, we show that these invariants are constants on branches, obtaining an anticyclotomic analogue of a similar result by Emerton-Pollack-Weston in the cyclotomic setting. This result allows us to spread results on the main conjecture from one form to all other forms in the family, obtaining new cases of the anticyclotomic main conjecture. This is a joint work with F. Castella and C.-H. Kim.

Never primitive points and the Lang Trotter Conjecture

Francesco Pappalardi

Institution: *Università di Roma 3*

ABSTRACT: La congettura di Lang - Trotter per punti primitivi predice un'espressione per la densità dei primi p per cui un fissato punto razionale (non di torsione) di una fissata curva ellittica definita su $\mathbb{Q}$ risulta un generatore della curva ridotta modulo p. Dopo aver fornito la definizione di tale densità in termini di rappresentazioni di Galois associate a punti di torsione della curva, racconteremo la scarna storia dei contributi alla congettura e forniremo esempi di famiglie di curve ellittiche per cui la congettura è verificata per ragioni banali. Si tratta della nozione di punti mai primitivi. Il caso delle curve ellittiche a moltiplicazione complessa verra discusso in maggiore dettaglio. Parte del lavoro è in collaborazione di N. Jones, G. Meleleo e P. Stevenhagen.

Arithmetic of Carlitz zeta values

Federico Pellarin

Institution: *Université de Saint-Etienne*

ABSTRACT: Carlitz zeta values are positive characteristic variants of the values of Riemann's zeta function. Since L. Carlitz and later, D. Goss, they are known to encode arithmetic properties of function fields in positive characteristic. We will give an overview of recent progress in the study of these and more general L-values and zeta values (analytic interpolations, zeroes, class number formula, finite variants etc.).

Heights bounds and anomalous intersections

Evelina Viada

Institution: *ETH Zürich*

ABSTRACT: I will present some joint works with S. Checcoli and F. Veneziano on the non density of anomalous intersections for some subvarieties of abelian varieties. The method uses several heights inequalities. In some cases we compute the constants, making the result explicit. I will also present some new question.

Heegner points and Iwasawa theory of supersingular elliptic curves

Stefano Vigni

Institution: *Università di Genova*

ABSTRACT: We explain how Heegner points on Shimura curves can be used to study the Iwasawa theory of a rational elliptic curve E over anticyclotomic $\mathbb{Z}_p$-extensions of imaginary quadratic fields, where p is a prime of supersingular reduction for E. This is based on joint work (partly in progress) with Mirela Ciperiani (Austin) and Matteo Longo (Padova).

The Selberg integral and a new pair-correlation function for the zeros of the Riemann zeta-function

Alessandro Languasco, Alberto Perelli, Alessandro Zaccagnini*

Institution: *Università di Parma - Dipartimento di Matematica e Informatica - Italia*

e-mail: alessandro.zaccagnini@unipr.it

2010 MSC: 11M26, 11N05

ABSTRACT: Let

$$J(x,\theta)=\int_x^{2x}|\psi(t)-\psi(t-\theta t)-\theta t|^2\,\mathrm{d}t$$

denote the "variance" of the primes in short intervals, where $\theta\in[0,1]$ and

$$F(X,T)=\sum_{\gamma_1,\gamma_2\in[0,T]}\frac{4X^{\mathrm{i}(\gamma_1-\gamma_2)}}{4+(\gamma_1-\gamma_2)^2}$$

denote Montgomery's pair-correlation function for the zeros of the Riemann zeta-function. We extend results of Goldston & Montgomery, Montgomery & Soundararajan and Chan, providing more accurate relations between hypothetical two-term asymptotic formulas for J and F (see [1]). Then we propose a new, more general pair-correlation function and discuss in detail its relevance to the distribution of prime numbers in short intervals, and give several applications (see [2], [3]).

References

[1] A. Languasco, A. Perelli, and A. Zaccagnini, *Explicit relations between pair correlation of zeros and primes in short intervals*, J. Math. Anal. Appl., **394** (2012), 761–771.

[2] A. Languasco, A. Perelli, and A. Zaccagnini, *An extension of the pair-correlation conjecture and applications*, Math. Res. Lett. (to appear). Arxiv preprint 1308.3934.

[3] A. Languasco, A. Perelli, and A. Zaccagnini, *An extended pair-correlation conjecture and primes in short intervals*, Trans. Amer. Math. Soc. (to appear). Arxiv preprint 1311.0597.

SHORT TALKS

Linear relations on families of powers of elliptic curves

Fabrizio Barroero* and Laura Capuano

Institution: *Scuola Normale Superiore di Pisa, Classe di Scienze*

e-mail: fbarroero@gmail.com

2010 MSC: 11G05, 11G50,11U09,14K05

ABSTRACT: In a recent work Masser and Zannier showed that there are at most finitely many complex numbers $\lambda \neq 0, 1$ such that the points $(2, \sqrt{2(2-\lambda)})$ and $(3, \sqrt{6(3-\lambda)})$ are simultaneously torsion on the Legendre elliptic curve E_λ of equation $y^2 = x(x-1)(x-\lambda)$. This is a special case of conjectures about Unlikely Intersections on families of abelian varieties, proved later in the two dimensional case by the same authors. As a natural higher dimensional extension, we considered the case of three points $(2, \sqrt{2(2-\lambda)})$, $(3, \sqrt{6(3-\lambda)})$ and $(5, \sqrt{20(5-\lambda)})$ and proved that there are at most finitely many $\lambda \neq 0, 1$ such that these three points satisfy two independent linear relations on E_λ. This is a special case of a more general result in the framework of the conjectures mentioned above.

Integer polynomials with small integrals

Danilo Bazzanella

Institution: *Politecnico di Torino - Dipartimento di Scienze Matematiche - Italia*

e-mail: danilo.bazzanella@polito.it

2010 MSC: 11C08, 11A41

ABSTRACT: The smart method of Gelfond–Shnirelman–Nair allows to obtain in elementary way a lower bound for the prime counting function $\pi(x)$ in terms of integrals of suitable integer polynomials. In this talk we present some properties of the class of integer polynomials relevant for the method.

References

[1] D. Bazzanella, *A note on integer polynomials with small integrals*, Acta Math. Hungar. **141** (2013), no. 4, 320–328.

[2] D. Bazzanella, *A note on integer polynomials with small integrals 2*, to appear.

Stabilizzazione in teoria di Iwasawa non abeliana

A. Bandini e F. Caldarola*

Institution: *Università della Calabria - Dipartimento di Matematica e Informatica - Italia*

e-mail: caldarola@mat.unical.it

2010 MSC: 11R23, 11R29.

ABSTRACT: Sia p un numero primo e K/k una $\mathbb{Z}_p$-estensione di un campo di numeri k. In [3] M. Ozaki studia la massima pro-p estensione non ramificata (non necessariamente abeliana) $\widetilde{L}/K$ di K e introduce a tal scopo i *moduli di Iwasawa superiori* $X^{(i)} := X^{(i)}(K/k),\ i \geqslant 1$ definiti per mezzo della serie centrale inferiore (*lower central series*, per $i = 1$ coincide con il modulo di Iwasawa classico $X(K/k)$ di K/k). Considerando i diversi livelli (o *layers*) k_n di una $\mathbb{Z}_p$-estensione, sono ben note le proprietà di stabilizzazione per l'ordine del modulo $X_n^{(1)}$ (isomorfo al gruppo delle classi di $k_n,\ n \geqslant 0$) e per il suo p-rango. In [1] viene prodotto un controesempio che mostra come una diretta generalizzazione ai moduli $X_n^{(i)}$ non sia possibile e si ottiene il seguente teorema di stabilizzazione che utilizza ipotesi più forti:

Teorema 1 *Sia $i \geqslant 1$ e $n \geqslant n_0$. Se $|X_n^{(j)}| = |X_{n+1}^{(j)}|$ per ogni $1 \leqslant j \leqslant i$, allora $X_n^{(j)} \simeq X_m^{(j)} \simeq X^{(j)}$ per ogni $1 \leqslant j \leqslant i$ e per tutti gli $m \geqslant n$.*

Presenteremo risultati analoghi anche per il p-rango degli $X_n^{(i)}$. Inoltre, in [2], si studiano stabilizzazione e capitolazione nel contesto classico (abeliano) per moduli come i *capitulation kernels* $H_{n,m} = \mathrm{Ker}\{i_{n,m} : A(k_n) \longrightarrow A(k_m)\}$ (dove $A(k_n)$ è il gruppo delle classi di k_n e $i_{n,m}$ è la mappa indotta dall'inclusione degli ideali) o $H_n = \bigcup_{m \geqslant n} H_{n,m}$; gli $H_{n,m}$ forniscono il primo esempio di non stabilizzazione nel contesto abeliano. Se il tempo lo consentirà, tratteremo brevemente anche altri moduli di Iwasawa e la loro relazione con le classiche congetture di Greenberg.

References

[1] A. Bandini e F. Caldarola, *Stabilization in non-abelian Iwasawa theory*, Acta Arith. **169** (2015), no. 4, 319–329.

[2] A. Bandini e F. Caldarola, *Stabilization for Iwasawa modules in $\mathbb{Z}_p$-extensions*, to appear in Rend. Semin. Mat. Univ. Padova.

[3] M. Ozaki, *Non-abelian Iwasawa theory of $\mathbb{Z}_p$-extensions*, J. Reine Angew. Math. **602** (2007), 59–94.

On the "almost" Pell's equation over polynomial rings

Fabrizio Barroero and Laura Capuano*

Institution: *Scuola Normale Superiore di Pisa - Italy*

e-mail: laura.capuano@sns.it

2010 MSC: 11G10, 14K15, 11U09, 11G50, 14K20

ABSTRACT: It is a classical result that, for any positive integer d not a perfect square, there exist integers x and $y \neq 0$ such that $x^2 - dy^2 = 1$. The analogous assertion for $D, X, Y \neq 0$ in $\mathbb{C}[t]$ with $X^2 - DY^2 = 1$ clearly requires that the degree of D should be even and in general solvability is no longer guaranteed if we only ask that D is not a perfect square in $\mathbb{C}[t]$.
In the talk we shall let $D(t) = D_\lambda(t)$ vary in a pencil. When $D_\lambda(t)$ has degree ≤ 4, it may be seen that for infinitely many complex λ there are nontrival solutions. On the other hand, this is not so when $D_\lambda(t)$ has degree 6. In this context, as an application of some results about Unlikely Intersections for certain families of abelian surfaces, Masser and Zannier proved that, if $D_\lambda(t) = t^6 + t + \lambda$, the Pell's equation is solvable nontrivially only for finitely many $\lambda \in \mathbb{C}$.
Here we consider a variant of this, namely the "almost" Pell equation $X^2 - DY^2 = P$, where $P(t) \in \overline{\mathbb{Q}}[t]$ is a polynomial of degree at most 2. When $D_\lambda(t)$ varies in the previous pencil, we have another finiteness result. This is a consequence of theorems on Unlikely Intersections for points on Jacobians of genus two curves.

Abelian Varieties over finite fields

Tommaso Centeleghe*, Jakob Stix

Institution: *University of Heidelberg - Department of Mathematics - Post Doc*

e-mail: tommaso.centeleghe@iwr.uni-heidelberg.de

2010 MSC: 11G15, 14K10

ABSTRACT: Let p be a prime number, $\mathbf{F}_q$ "the" finite field of characteristic p and order $q = p^e$, and AV_q the category of abelian varieties over $\mathbf{F}_q$. In an ongoing project the authors study the problem of concretely describing AV_q and certain full subcategories in terms of linear algebra data. The general method is inspired by Waterhouse's thesis (cf. [3]) and builds on the construction of a lattice $T(A)$ functorially attached to any object A of AV_q and equipped with the extra structure given by the action of a certain pro-ring $\mathcal{R}_q$. The first result obtained (cf. [1]) pertains only the case $q = p$ and asserts the existence of an equivalence between the full subcategory $\mathsf{AV}_p^{\mathrm{com}} \subset \mathsf{AV}_p$ of varieties whose Frobenius operator avoids the eigenvalues $\pm\sqrt{p}$, and the category of pairs (T, F) consisting of a finite free $\mathbf{Z}$-module T and a linear operator $F : T \to T$ satisfying two axioms easy to state. This generalizes the $q = p$ case of theorem of Deligne (cf. [2]), who gave an analogous

description of the subcategory $\mathsf{AV}_q^{\mathrm{ord}} \subset \mathsf{AV}_q$ of ordinary abelian varieties. While the main ingredient in Deligne's proof is Serre-Tate theory of canonical liftings of ordinary abelian varieties, the method employed by the authors relies on the Gorenstein property of the orders generated by a Weil p-numbers and it complex conjugate, and avoids lifting objects to characteristic zero.

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Equivariant epsilon constant conjectures for weakly ramified extensions

Alessandro Cobbe*, Werner Bley

Institution: *Universität der Bundeswehr München - Institut für Theoretische Informatik, Mathematik und Operations Research - Germany*

e-mail: alessandro.cobbe@unibw.de

2010 MSC: 11Y40

ABSTRACT: We study the local epsilon constant conjecture as formulated by Breuning in [2]. This conjecture fits into the general framework of the equivariant Tamagawa number conjecture (ETNC) and should be interpreted as a consequence of the expected compatibility of the ETNC with the functional equation of Artin-L-functions. Let $K/\mathbb{Q}_p$ be unramified. Under some mild technical assumption we prove Breuning's conjecture for weakly ramified abelian extensions N/K with cyclic ramification group. As a consequence of Breuning's local-global principle we obtain the validity of the global epsilon constant conjecture as formulated in [1] and of Chinburg's $\Omega(2)$-conjecture as stated in [3] for certain infinite families F/E of weakly and wildly ramified extensions of number fields.

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Sieve functions in almost all short intervals

Giovanni Coppola* and Maurizio Laporta

Institution: *Università di Salerno - Salerno - Italia*

e-mail: giovanni.coppola@unina.it

2010 MSC: 11N37, 11N25

ABSTRACT: An arithmetic function f is called a *sieve function of range* Q, if it is the convolution product of the constantly 1 function and g such that $g(q) \ll_\varepsilon q^\varepsilon$, $\forall \varepsilon > 0$, for $q \leqslant Q$, and $g(q) = 0$ for $q > Q$, i.e.

$$f(n) := \sum_{\substack{q|n \\ q\leq Q}} g(q).$$

For example, the GPY truncated divisor sum Λ_R involves sieve functions of range R. In a joint work with Maurizio Laporta ([1], [2]), we have started the study of the distribution of f in short intervals by analyzing its so-called *weighted Selberg integral*

$$J_{w,f}(N,H) := \sum_{N<x\leq 2N} \Big| \sum_{x-H\leq n\leq x+H} w(n-x)f(n) - M_f(x,w)\Big|^2,$$

where w is a complex valued *weight*, that is bounded and supported in $[-H,H]$ with $H = o(N)$, as $N \to \infty$, while $M_f(x,w) := \sum_a w(a) \sum_{q\leq Q} g(q)/q$ is the expected (short intervals) *mean value* of the weighted f (see [1] for more general considerations when f has its Dirichlet series in the Selberg Class). We'll give a short panorama of our methods to estimate $J_{w,f}(N,H)$, including the recent applications of our results on the distribution of f over short *arithmetic bands* [3]

$$\bigcup_{1\leq a\leq H} \{n \in (N,2N] : n \equiv a \,(\mathrm{mod}\, m)\}.$$

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Manin-Mumford conjecture in Function Field Arithmetic

Luca Demangos

Institution: *Instituto de Matematicas, UNAM - Unidad Cuernavaca, Mexico*

e-mail: l.demangos@gmail.com

2010 MSC: 11G09,11G25

ABSTRACT: We try to adapt the Manin-Mumford conjecture to the setting of G. Anderson's $T-$modules. We propose a statement in such a context and we trace a proof strategy for a special class of $T-$modules (the abelian and uniformizable ones) based on U. Zannier's and J. Pila's recently developped techniques of Diophantine Geometry.

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Automorphisms of non-split Cartan modular curves

Valerio Dose

Institution: *Università di Roma "Tor Vergata" - Dipartimento di Matematica - Italia*

e-mail: dose@mat.uniroma2.it

2010 MSC: 14G35,11G05

ABSTRACT: Modular curves are Riemann surfaces whose points (more precisely all but finitely many of them) parametrize elliptic curves over $\mathbb{C}$. Usually, modular curves can be constructed as the compactification of quotients of the upper-half complex plane $\mathcal{H} = \{\tau \in \mathbb{C} \text{ s. t. } \mathrm{Im}(\tau) > 0\}$, by a certain subgroup Γ of $\mathrm{SL}_2(\mathbb{Z})$. They also have the structure of projective algebraic curve, in general with equations over some cyclotomic field, and often over the field $\mathbb{Q}$ of rational numbers, depending on the group structure of Γ. The upper-half complex plane $\mathcal{H}$ has a rich automorphism group, namely $\mathrm{SL}_2(\mathbb{R})/\{\pm\mathrm{Id}\}$, whose elements acts on $\mathcal{H}$ as Möbius transformations. Given a modular curve X, **one can ask if all its automorphisms are induced by some automorphism of $\mathcal{H}$**. This is an interesting question because automorphisms of X induced by automorphisms of $\mathcal{H}$ preserve important information about the elliptic curves parametrized by the points of X. Thus, a negative answer to the question above can lead to unexpected symmetries in the family of elliptic curves parametrized by X. When the genus of X exceeds 1, very few examples of this phenomenon are known. Furthermore, these exceptional occurrences

are often source of exceptional behavior in the framework of **Serre's uniformity conjecture** ([3]), which is an important statement about Galois representations attached to elliptic curves over $\mathbb{Q}$ and it is equivalent to assert that certain families of modular curves do not have points with coordinates in $\mathbb{Q}$, with some limited exceptions. In all of the known exceptions, the presence of these points can be explained by the existence of some exceptional automorphism of the modular curve. The last significant case to understand, in the effort to prove Serre's uniformity conjecture, concerns the modular curves $X_{ns}^{+}(p)$ associated to the normalizer of a **non-split Cartan subgroup of $\mathbf{GL}_2(\mathbb{Z}/p\mathbb{Z})$**, where p is a prime number. We made some progress towards the determination of the automorphism group of modular curves $X_{ns}(p)$ associated to a non-split Cartan subgroup of $\mathrm{GL}_2(\mathbb{Z}/p\mathbb{Z})$, for almost all p ([1]). We also related Serre's conjecture to the lack of exceptional automorphisms of $X_{ns}(p)$. In a previous work ([2]), we computed explicitly equations and the automorphism group of $X_{ns}(11)$, proving the existence, in this case, of an exceptional automorphism.

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About nodal sets of eigenfunctions of the Laplacian on the torus

Riccardo W. Maffucci

Institution: *King's College London - Department of Mathematics - UK*

e-mail: riccardo.maffucci@kcl.ac.uk

2010 MSC: 11Hxx

ABSTRACT: We consider various aspects of nodal sets of eigenfunctions of the Laplacian on the torus in two and three dimensions.
We study the variance of the number of nodal intersections with a straight line in two dimensions. We bound the variance in case of a line with rational slope. Moreover, we bound the variance for all straight lines along certain sequences of the radius m. We also prove that a sharper upper bound for the variance is obtained if we assume a conjecture about lattice points on small arcs.
A natural continuation of this problem is the variance for nodal intersections with a straight line on the three dimensional torus. Another problem we want to study is the variance of the volume of the nodal set in the three dimensional case.

Algebraic equations with lacunary polynomials and the Erdős-Rényi conjecture

Vincenzo Mantova

Institution: *Scuola Normale Superiore - Classe di Scienze - Italia*

e-mail: vincenzo.mantova@sns.it

2010 MSC: 11C08, 12E05, 12Y05, 14G05, 14J99, 11U10

ABSTRACT: In 1949, Rényi and Erdős [1] independently conjectured that given a polynomial $g(X)$ over the complex numbers, if we have a bound on the number of non-zero terms of the square of $g(X)$, then there is a bound on the number of terms of $g(X)$ itself. This was proved by Schinzel [3], actually with any power in place of the square, and he asked whether the same is true for the composition $f(g(X))$, where $f(Y)$ is another given polynomial. This was finally proved by Zannier [4]. In a joint work with C. Fuchs and U. Zannier [3], we extend the result to the following more general case: if $g(X)$ is the root of a polynomial $F(Y)$ whose coefficients are themselves polynomials in X with a bounded number of terms, then $g(X)$ is at least the ratio of two polynomials with a bounded number of terms. This can be shown to imply the previous statements. The result has also several implications regarding irreducibility problems and integral points.

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On the structure of Shafarevich-Tate group associated to a higher weight modular form

Masoero Daniele

Institution: *Università di Genova - Dipartimento di Matematica*

e-mail: masoero@dima.unige.it

2010 MSC: 14C25, 11F11

ABSTRACT: Classical results due to Kolyvagin and dating back to the mid '80s describe the structure of Shafarevich-Tate groups of rational elliptic curves. In this talk, based on work in progress, I will describe analogous results for the Shafarevich-Tate group associated, following Nekovàř's recipe, with a higher weight modular form.

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Statistics for biquadratic covers of the projective line over finite fields

Elisa Lorenzo, Giulio Meleleo*, Piermarco Milione

Institution: *Università degli Studi "Roma Tre" - Dipartimento di Matematica e Fisica - Italia*

e-mail: meleleo@mat.uniroma3.it

2010 MSC: 11G20,14H05,11M50

ABSTRACT: We study the distribution of the traces of the Frobenius endomorphism of genus g curves which are quartic non-cyclic covers of $\mathbb{P}^1_{\mathbb{F}_q}$, as the curve varies in an irreducible component of the moduli space. We show that for q fixed, the limiting distribution of the trace of Frobenius equals the sum of $q+1$ independent random discrete variables. We also show that when both g and q go to infinity, the normalized trace has a standard complex Gaussian distribution. Finally, we extend part of these computations to the general case of arbitrary covers of $\mathbb{P}^1_{\mathbb{F}_q}$ with Galois group isomorphic to r copies of $\mathbb{Z}/2\mathbb{Z}$. For $r=1$, we recover the already known hyperelliptic case. The preprint of this work can be found on arXiv (http://arxiv.org/abs/1503.03276).

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On some questions about weird numbers

Giuseppe Melfi

Institution: *Université de Neuchâtel - Faculté de Sciences Economiques - Svizzera*

e-mail: Giuseppe.Melfi@unine.ch

2010 MSC: 11A25; 11B83

ABSTRACT: A weird number is a positive integer n whose sum of its proper divisors exceeds n but no sums of distinct proper divisors equals n.
I will survey some known results about weird numbers. For example there are infinitely many weird numbers and indeed their asymptotic density is positive.
I will speak about some old questions about weird numbers raised by Benkoski and Erdős in the '70s.
One of the most intriguing open questions is the existence of odd weird numbers. All known weird numbers are even, and recent computations only showed that there are no odd weird numbers up to 10^{21}. Erdős offered 10$ for the first example of an odd weird number and 25$ for the proof that no odd weird number exists.
If w is a weird number and $p > \sigma(w)$, then wp is a weird number. As a consequence of this elementary property, *primitive* weird numbers are defined as those weird numbers that are not a multiple of another weird number.
In a recent paper I showed that under the assumption of the Cramér conjecture, or even of much weaker conjectural assumptions on the distribution of primes in short intervals, there exist infinitely primitive weird numbers, another problem raised by Erdos and Benkoski.

Recent results about the prime ideal theorem

Giuseppe Molteni

Institution: *Università statale di Milano - Dipartimento di Matematica - Italia*

e-mail: giuseppe.molteni1@unimi.it

2010 MSC: 11R42, 11Y40

ABSTRACT: Let $\psi_{\mathbb{K}}$ be Chebyshev's function of a number field $\mathbb{K}$. Let also $\psi_{\mathbb{K}}^{(1)}(x) := \int_0^x \psi_{\mathbb{K}}(t)\,\mathrm{d}t$ and $\psi_{\mathbb{K}}^{(2)}(x) := 2\int_0^x \psi_{\mathbb{K}}^{(1)}(t)\,\mathrm{d}t$, which are smoothed versions of $\psi_{\mathbb{K}}$. Assuming GRH we prove explicit inequalities for $|\psi_{\mathbb{K}}(x) - x|$, $|\psi_{\mathbb{K}}^{(1)}(x) - \frac{x^2}{2}|$ and $|\psi_{\mathbb{K}}^{(2)}(x) - \frac{x^3}{3}|$. Two results about the existence of ideals having small norms and about the computation of the residue of Dedekind's zeta function are also given. This is an account of a joint work with Loïc Grenié: [1], [2].

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Carries and the arithmetic progression structure of sets

Francesco Monopoli

Institution: *Università degli Studi di Milano - Dipartimento di Matematica - Italia*

e-mail: francesco.monopoli@unimi.it

2010 MSC: 11P70

ABSTRACT:

If we want to represent integers in base m, we need a set A of digits, which needs to be a complete set of residues modulo m. When adding two integers with last digits $a_1, a_2 \in A$, we find the unique $a \in A$ such that $a_1 + a_2 \equiv a \bmod m$, and call $(a_1 + a_2 - a)/m$ the carry. Carries occur also when addition is done modulo m^2, with A chosen as a set of coset representatives for the cyclic group $\mathbb{Z}/m\mathbb{Z} \subseteq \mathbb{Z}/m^2\mathbb{Z}$. It is a natural to look for sets A which minimize the number of different carries. In a recent paper, Diaconis, Shao and Soundararajan proved that, when $m = p$, p prime, the only set A which induces two distinct carries, i. e. with $A + A \subseteq \{x, y\} + A$ for some $x, y \in \mathbb{Z}/p^2\mathbb{Z}$, is the arithmetic progression $[0, p-1]$, up to certain linear transformations. We present a generalization of

the result above to the case of generic modulus m^2, and show how this is connected to the uniqueness of the representation of sets as a minimal number of arithmetic progression of same difference. (Joint work with Imre Z. Ruzsa)

Periodic representations and simultaneous approximations of cubic irrationalities

Nadir Murru

Institution: *University of Turin - Department of Mathematics - Italy*

e-mail: nadir.murru@unito.it

2010 MSC: 11J70, 11K60

ABSTRACT: In 1839 Hermite posed to Jacobi the problem of generalizing the construction of continued fractions to higher dimensions. In particular, he asked for a method of representing algebraic irrationalities by means of periodic sequences of integers that can highlight algebraic properties and possibly provide rational approximations. Hermite especially focused the attention on cubic irrationalities. Continued fractions completely solve this problem for quadratic irrationalities, but the problem for algebraic numbers of degree $\geqslant 3$ is still open. Here, we present an approach to this problem developed in a recent paper of the author [1], where a periodic representation for all cubic irrationalities is provided. The periodic representation is provided by means of ternary continued fractions exploiting properties of linear recurrence sequences and generalizing classical Rédei rational functions. The periodic representation is obtained starting from the knowledge of minimal polynomial of the involved cubic irrational. Thus, Hermite problem still remains open, since an algorithm defined over all real numbers is not defined.
Moreover, we show rational approximations that arise from the periodic representation by means of powers of 3×3 matrices. These approximations can be generalized by means of regular representation of cubic extensions. In this way, we found a generalization of Khovanskii matrices (which are only used for approximations of cubic roots) that allows to provide rational approximations given any cubic irrational. The method produce rational approximations in a fast way with more accuracy (considering same size of denominators) than approximations provided by iterative methods, as Newton and Hallley ones.

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Preperiodic points of rational maps defined over a global field in terms of good reduction

Jung Kyu Canci and Laura Paladino*

Institution: *University of Pisa - Department of Mathematics - Italy*

e-mail: paladino@mail.dm.unipi.it

2010 MSC: 37P05, 11D45

ABSTRACT: Let K be a global field. We denote by p its characteristic. Let D be the degree of K over the base field, that is $\mathbb{Q}$ when $p = 0$ and $\mathbb{F}_p(t)$ when $p > 0$. We denote by ϕ an endomorphism of the projective line defined over K and by d its degree. A point $P \in \mathbb{P}_1(K)$ is called *periodic* for ϕ if there exists an integer $n > 0$ such that $\phi^n(P) = P$. We say that P is a *preperiodic* point for ϕ if its (forward) orbit $O_\phi(P) = \{\phi^n(P) \mid n \in \mathbb{N}\}$ contains a periodic point, i. e. it is finite. We show a bound for the cardinality of $O_\phi(P)$ depending only on D, p and the number of places of bad reduction of ϕ. Furthermore, we show a bound for the cardinality of the set of K–rational preperiodic points for ϕ, depending on D, d, p and the number of places of bad reduction of ϕ. The results are completely new in the function fields case and they improve the ones known in the number fields case.

Zeros of combinations of Euler products for $\sigma > 1$

Mattia Righetti

Institution: *Università degli Studi di Genova - Dipartimento di Matematica - Italia*

e-mail: righetti@dima.unige.it

2010 MSC: 11M41, 11M26

ABSTRACT: It is well known that linear combinations of L-functions may not satisfy the Riemann Hypothesis. For example, in 1936 Davenport and Heilbronn showed that the Hurwitz zeta function $\zeta(s, a)$ has infinitely many zeros for $\sigma > 1$ when a is transcendental or rational with $a \neq \frac{1}{2}$, and Cassels showed that the same is true when a is irrational algebraic. Note that when a is rational, $\zeta(s, a)$ is a linear combination of Dirichlet L-functions. Another example is given by the Epstein zeta function $\zeta(s, Q)$ associated to a positive definite binary quadratic form Q. Also for these functions Davenport and Heilbronn showed that they have infinitely many zeros for $\sigma > 1$ when the class number $h(D)$ of the quadratic field $\mathbb{Q}[\sqrt{D}]$, where D is the discriminant of Q, is > 1. Again, note that if $h(D) > 1$ then $\zeta(s, Q)$ may be written as a non-trivial linear combination of Hecke L-functions.

The talk will be on recent developments in this direction, namely we will show for many classes of L-functions with an Euler product that non-trivial linear and non-linear combinations have always zeros for $\sigma > 1$.

Moreover, we will discuss and show some results on the related problem about the distribution of the real parts of such zeros for $\sigma > 1$.

Funzioni L p-adiche e zero banali per forme di Siegel

Giovanni Rosso

Institution: *KU Leuven - Department of Mathematics - Belgium*

e-mail: gvnros@gmail.com

2010 MSC: 11F33, 11F67, 11F85,

ABSTRACT: Sia p un numero primo fissato e sia E una curva ellittica con cattiva riduzione moltiplicativa in p. A partire dal fondamentale lavoro di Greenberg e Stevens sulla congettura di Mazur, Tate e Teitelbaum sulla derivata della funzione L p-adica di E, Greenberg e Benois hanno formulato una congettura sul comportamento delle funzioni L p-adiche $L_p(M,s)$ per motivi M che presentano dei così detti *zeri banali*. Questa congettura esprime l'ordine e il coefficiente direttore della funzione L p-adica in $s=0$ in termine di semplici invarianti legati a M.
In questo seminario spiegherò come questa congettura può essere trattata nel caso di forme di Siegel ordinarie e Steinberg in p e come il recente lavoro di Z. Liu sugli operatori di Maaß-Shimura p-adici e sulle forme di Siegel quasi surconvergenti permetta la generalizzazione di questo risultato a forme di pendenza finita.

Sulla razionalità dei periodi integrali: il caso compatto

M. Greenberg e M. A. Seveso*

Institution: *Università degli studi di Milano - Dipartimento di matematica Federigo Enriques - Italia*

e-mail: seveso.marco@gmail.com

2010 MSC: 11F67

ABSTRACT: Sia $\mathbf{G}_{/\mathbb{Q}}$ un gruppo riduttivo su $\mathbb{Q}$ tale che $\mathbf{G}(\mathbb{Q}) \subset \mathbf{G}(\mathbb{A}_f)$ è discreto e $(\mathbf{G}/\mathbf{Z}_{\mathbf{G}})(\mathbb{R})$ è compatto. Dato un morpfismo di gruppi algebrici

$$\eta : \mathbf{H} \to \mathbf{G}$$

tale che $\mathbf{H}$ soddisfa una simile condizione con $\mathbf{Z}_{\mathbf{G}}$ rimpiazzato da $\mathbf{Z}_\eta := \eta^{-1}(\mathbf{Z}_{\mathbf{G}}) \cap \mathbf{Z}_{\mathbf{H}}$, possiamo definire un periodo integrale

$$I : L^2\left(\mathbf{G}(\mathbb{A})/\mathbf{G}(\mathbb{Q}), \omega\right) \to \mathbb{C}$$

mediante la formula

$$I(f) := \int_{\mathbf{Z}_\eta(\mathbb{A})\backslash\mathbf{H}(\mathbb{A})/\mathbf{H}(\mathbb{Q})} f(\eta(h))\,\omega^{-1}(\eta(h))\,d\mu_{\mathbf{Z}_\eta(\mathbb{A})\backslash\mathbf{H}(\mathbb{A})/\mathbf{H}(\mathbb{Q})}.$$

L'interesse nei confronti di questo tipo di funzionali risiede nel fatto che essi sono legati, in vari contesti, a formule per il valore speciale delle funzioni L nel punto critico centrale. Forniremo dunque degli esempi concreti ed esporremo un risultato che mostra come questi integrali sono algebrici. Più precisamente, data una $\mathbb{Q}$-algebra R, è possibile introdurre una nozione di forme automorfe a coefficienti in R con la proprietà che, se $R = \mathbb{C}$, tale nozione restituisce lo spazio delle forme automorfe finite, che è denso in $L^2(\mathbf{G}(\mathbb{A})/\mathbf{G}(\mathbb{Q}), \omega)$. Si ottiene in questo modo un funtore e la restrizione di I alle forme automorfe finite si estende un morfismo di funtori a valori in $\mathbf{A}^1$.

On the canonical height on endomorphisms ring of a vector space over a global function field

Valerio Talamanca

Institution: *Università Roma Tre, Dipartimento di Architettura, Rome, Italy*

e-mail: valerio@mat.uniroma3.it

2010 MSC:

ABSTRACT: In this talk we will discuss the construction of a canonical height on the endomorphisms ring of a vector space E over a global function field k. The height constructed satisfies a limit formula, analogous to the formula satisfied by the canonical height attached to an ample line bundle on an abelian variety, with respect to any heights on E given by the assignment of a pure adelic vector bundle structure on E. As a consequence we will construct the canonical height on any étale k-algebra.

Uniformity results in Diophantine Geometry

Amos Turchet*, Kenneth Ascher

Institution: *Chalmers University of Technology and Göteborg University - Department of Mathematical Sciences - Göteborg - Sweden*

e-mail: tamos@chalmers.se

2010 MSC: 14G05, 11D45

ABSTRACT: In 1997 Caporaso, Harris and Mazur proved that Lang Conjecture (i.e. rational points in general type varieties are not Zariski dense) implies that the number of rational point in curves of genus > 1 are not only finite (Falting's Theorem) but uniform; in particular there exists a bound for their number depending only on the genus and on the base field. This result has been extended to surfaces of general type by work of Hassett. Analogous problems have been treated for (stably) integral points - introduced by Abramovich - for elliptic curves and principally polarised abelian varieties, where uniformity has been proved to hold by work of Abramovich and Abramovich-Matzuki. I will

report on a work-in-progress project, joint with Kenneth Ascher, aiming to extend the results for integral points to all log general type surfaces.

Euler Characteristic of Selmer groups over global function fields

Andrea Bandini, Maria Valentino*, Stefano Vigni

Institution: *Università della Calabria - Dipartimento di Matematica e Informatica - Italia*

e-mail: maria.valentino84@gmail.com

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ABSTRACT: Let $\ell \in \mathbb{Z}$ be a prime and let G be a profinite ℓ-adic Lie group with no elements of order ℓ and of finite dimension. Let M be a G-module and consider the following properties:

1. $H^i(G, M)$ is finite for any $i \geqslant 0$;

2. $H^i(G, M) = 0$ for all but finitely many i.

If a G-module M verifies **1** and **2**, we define the *Euler characteristic* of M as

$$\chi(G, M) := \prod_{i \geqslant 0} |H^i(G, M)|^{(-1)^i} .$$

For a field F let A/F be an abelian variety and, for any extension L/F let $Sel_A(L)_\ell$ be the ℓ-part of the Selmer group of A over L. When G is the Galois group of a field extension K/F, the study of the Euler characteristic $\chi(G, Sel_A(K)_\ell)$ is a first step towards understanding the relation (predicted by the Iwasawa Main Conjecture) between a characteristic element for $Sel_A(K)_\ell$ and a suitable ℓ-adic L-function.

During the talk we will see Euler characteristic formulas for the Selmer group in the function field case. Let F be a global function field of characteristic $p > 0$ and let K/F be an ℓ-adic Lie extension (ℓ any prime) with Galois group G and unramified outside a finite and nonempty set S of primes of F.
For the $\ell \neq p$ case we shall provide two formulations for $\chi(G, Sel_A(K)_\ell)$. The first mainly depends on the cohomology of torsion points, while the second involves more directly the Tate-Shafarevich group. Therefore, the latter suggests a connection with special values of L-functions and the Birch and Swinnerton-Dyer conjecture.
We will see an Euler characteristic formula also for the $\ell = p$ case. In this instance the calculations mostly involve flat cohomology of the group scheme $A[p^\infty]$.

If time allows, we will describe some preliminary results on the Hecke algebra for Drinfeld modular forms. This matter can be seen as a first step towards Hida theory for global function fields. We believe that, as in the number field setting, the generalization of Hida theory to the finite characteristic case should provide an interesting link between arithmetic objects and analytic ℓ-adic L-functions.

The anticyclotomic Iwasawa main conjecture for elliptic curves

Massimo Bertolini, Rodolfo Venerucci*

Institution: *Fakultät für Mathematik, Universität Duisburg-Essen*

e-mail: rodolfo.venerucci@uni-due.de

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ABSTRACT: Let E be an elliptic curve defined over $\mathbf{Q}$, let $K/\mathbf{Q}$ be an imaginary quadratic field, and let p be a prime. The field K has a unique $\mathbf{Z}_p$-extension K_∞/K which is Galois and pro-dihedral over $\mathbf{Q}$, called the anticyclotomic $\mathbf{Z}_p$-extension. For every $n \in \mathbf{N}$, denote by K_n/K the cyclic sub-extension of K_∞/K of degree p^n. To every character $\chi : \mathrm{Gal}(K_n/K) \to \overline{\mathbf{Q}}_p^*$ is associated the complex L-function $L(E/K, \chi, s)$ of E/K twisted by χ. Roughly speaking, the anticyclotomic Iwasawa main conjectures — formulated in different settings by Perrin-Riou and by Bertolini–Darmon — relate the special values of $L(E/K, \chi, s)$ at $s = 1$, for varying χ, to the arithmetic of E over K_∞. More precisely, depending on the sign in the functional equation satisfied by $L(E/K, \chi, s)$, the algebraic part of $L(E/K, \chi, 1)$ or of $L'(E/K, \chi, 1)$ is related to the structure of the χ-part of the Selmer group of E/K_n. In this talk I will report on a joint work with Massimo Bertolini, in which we prove the anticyclotomic main conjectures for elliptic curves at primes of ordinary reduction.

Some explicit cases of the Torsion Anomalous Conjecture

Sara Checcoli, Francesco Veneziano*, Evelina Viada

Institution: *Università di Basilea - Dipartimento di matematica e informatica - Svizzera*

e-mail: francesco.veneziano@unibas.ch

2010 MSC: 11G50, 14G40

ABSTRACT: The Torsion Anomalous Conjecture states that an algebraic variety V in a semi-abelian variety contains finitely many maximal torsion anomalous subvarieties. In a paper with Sara Checcoli and Evelina Viada we prove some new cases of this conjecture for V a weak-transverse variety in a product of elliptic curves. Our main result provides a totally explicit bound for the Néron-Tate height of all V-torsion anomalous points of relative codimension one in the non CM case and an analogous effective result in the CM case. As an application, we obtain new explicit results in the context of the effective Mordell-Lang Conjecture; in particular we bound the Néron-Tate height of the rational points of an explicit family of curves of increasing genus.