Geometry, arithmetic, and dynamics of S-adic systems I

joint work with P. Arnoux, V. Berthé, M. Minervino
W. Steiner, J.Thuswaldner, R. Yassawi

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Renormalization in dynamics

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Part I: one-sided case

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Renormalization in dynamics

The Pisot substitution conjecture

Let σ be a Pisot irreducible substitution

$$\begin{array}{ccc} X_\sigma & \xrightarrow{\Sigma \text{ shift}} & X_\sigma \\ \downarrow & & \downarrow \\ G & \xrightarrow[g \mapsto ag]{} & G \end{array}$$

- (X_σ, Σ) is measure-theoretically isomorphic to a translation on a compact abelian group
- (X_σ, Σ) has pure discrete spectrum

Substitutive structure + Algebraic assumption (Pisot)

= Order (discrete spectrum)

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The Pisot substitution Conjecture dates back to the 80's

[Bombieri-Taylor, Rauzy,Thurston]

The conjecture is proved for two-letter alphabets

[Host, Barge-Diamond, Hollander-Solomyak]

In the framework of β-numeration

Theorem [Barge]

- If σ is a Pisot irreducible substitution that is injective on initial letters, and constant on final letters, then (X_σ, Σ) has pure discrete spectrum

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- If β is a Pisot number, the tiling dynamical system associated with the β-substitution has pure discrete spectrum ($\mathbb{R}$-action, suspension flow)

- The arithmetical coding of the hyperbolic solenoidal automorphism associated with the companion matrix of the minimal polynomial of any Pisot number is a.e. one-to-one

Pisot substitutions

Let σ be a primitive substitution. Then (X_σ, Σ) is uniquely ergodic, linearly recurrent, it has bounded factor complexity

Pisot substitution σ is primitive and its Perron–Frobenius eigenvalue is a Pisot number

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Fact Symbolic dynamical systems (X_σ, Σ, μ) generated by Pisot substitutions have bounded symbolic discrepancy

$$\Delta_N = \max_{i \in \mathcal{A}} \left| |u_0 u_1 \dots u_{N-1}|_i - N \cdot f_i \right| \leq C\, N, \quad \forall N, \quad \forall u \in X_\sigma$$

$$f_i = \lim_{N \to \infty} \frac{|u_0 \cdots u_{N-1}|_i}{N} = \mu[i] = \text{frequency of occurrence of the letter } i$$

Bounded ergodic averages

Cylinders associated with letters are Bounded Remainder Sets

Why taking the Pisot assumption?

f is a coboundary iff its ergodic sums are bounded

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Theorem [Gottschalk-Hedlund] Let X be a compact metric space and $T\colon X \to X$ be a minimal homeomorphism. Let $f\colon X \to \mathbb{R}$ be a continuous function. Then f is a coboundary

$$f = g - g \circ T$$

for a continuous function g if and only if there exists $C > 0$ such that for all N and all x

$$|\sum_{n=0}^{N} f(T^n(x))| < C$$

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Take $\quad f = \mathbf{1}_{[a]}(x) - f_a \mathbf{1} \quad \rightsquigarrow \quad f = g - g \circ \Sigma$

$$\exp^{2i\pi g \circ \Sigma} = \exp^{2i\pi f_a} \exp^{2i\pi g}$$

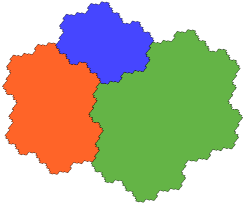
$\exp^{2i\pi g}$ is a continuous eigenfunction associated with the eigenvalue $\exp^{2i\pi f_a}$ $\rightsquigarrow$ Rotation factor

Tribonacci dynamics and Tribonacci Kronecker map

$$\sigma : 1 \mapsto 12,\ 2 \mapsto 13,\ 3 \mapsto 1$$

Theorem [Rauzy'82] The symbolic dynamical system (X_σ, Σ) is measure-theoretically isomorphic to the translation R_β on the two-dimensional torus $\mathbb{T}^2$

$$R_\beta : \mathbb{T}^2 \to \mathbb{T}^2,\ x \mapsto x + (1/\beta, 1/\beta^2)$$



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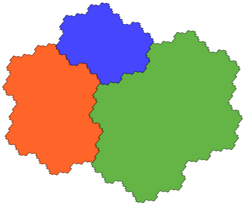

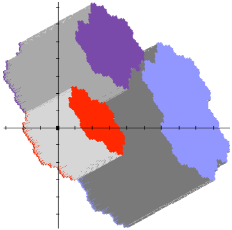

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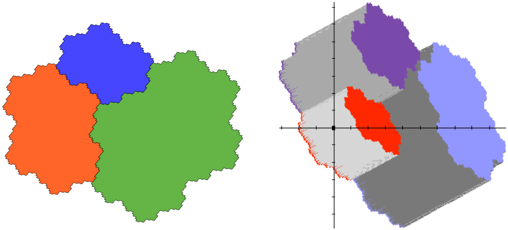

Markov partition for the toral automorphism $\begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$

How to reach nonalgebraic parameters?

Theorem [Rauzy'82]

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- We want to find symbolic realizations for Kronecker maps (toral translations)
- We want to reach nonalgebraic parameters
- We consider not only one substitution

How to reach nonalgebraic parameters?

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- We want to find symbolic realizations for a.e. Kronecker maps (toral translations)
- We want to reach nonalgebraic parameters by considering convergent products of matrices
- We consider not only one substitution but a sequence of substitutions S-adic formalism Non-stationary dynamics

Continued fractions algorithm $\rightsquigarrow$ renormalization

S-adic expansions and non-stationary dynamics

Definition An infinite word ω is said S-adic if there exist

- a set of substitutions $\mathcal{S}$
- an infinite sequence of substitutions $(\sigma_n)_{n\geq 1}$ with values in $\mathcal{S}$

such that

$$\omega = \lim_{n\to+\infty} \sigma_1 \circ \sigma_2 \circ \cdots \circ \sigma_n(0)$$

The terminology comes from Vershik adic transformations
Bratteli diagrams

S stands for substitution, adic for the inverse limit
powers of the same substitution= partial quotients

Dictionary

S-adic description of a minimal symbolic dynamical system $\rightleftharpoons$ multidimensional continued fraction algorithm that governs its letter frequency vector/ invariant measure

- S-adic expansion
- Unique ergodicity
- Linear recurrence
- Balance and Pisot properties
- Two-sided sequences of substitutions
- Shift on sequences of substitutions

- Continued fraction
- Convergence
- Bounded partial quotients
- Strong convergence
- Natural extension
- Continued fraction map

Which continued fraction algorithms?

We focus here on two algorithms

- Arnoux-Rauzy algorithm

$$(a, b, c) \mapsto (a - (b + c), b, c) \text{ if } a \geq b + c$$

- Brun algorithm

$$(a, b, c) \mapsto \text{Sort}(a, b, c - b) \text{ if } a \leq b \leq c$$

Which continued fraction algorithms?

We focus here on two algorithms

- Arnoux-Rauzy algorithm
  - Defined on a set of zero measure
  - Coding plus projection of an exchange of 6 intervals on the circle
  - They code particular systems of isometries (thin type) (pseudogroups of rotations) [Arnoux-Yoccoz, Novikov, De Leo-Dynnikov, Gaboriau-Levitt-Paulin, etc.]
  - A geometric context: natural suspension flow
  - Invariant measure, simplicity of the Lyapunov exponent [Avila-Hubert-Skripchenko]

- Brun algorithm
  - Invariant measure, natural extension, Lyapunov exponents, exponential convergence are well-known
  - Flow?

S-adic Pisot dynamics

Theorem [B.-Steiner-Thuswaldner]

- For almost every $(\alpha, \beta) \in [0, 1]^2$, the translation by (α, β) on the torus $\mathbb{T}^2$ admits a natural symbolic coding provided by the S-adic system associated with Brun multidimensional continued fraction algorithm applied to (α, β)
- For almost every Arnoux-Rauzy word, the associated S-adic system has pure discrete spectrum

Arnoux-Rauzy words

$$\sigma_1: \begin{array}{ll} 1 & \mapsto 1 \\ 2 & \mapsto 21 \\ 3 & \mapsto 31 \end{array} \qquad \sigma_2: \begin{array}{ll} 1 & \mapsto 12 \\ 2 & \mapsto 2 \\ 3 & \mapsto 32 \end{array} \qquad \sigma_3: \begin{array}{ll} 1 & \mapsto 13 \\ 2 & \mapsto 23 \\ 3 & \mapsto 3 \end{array}$$

$$\omega = \lim_{n\to\infty} \sigma_{i_0}\sigma_{i_1}\cdots\sigma_{i_n}(1)$$

and every letter in $\{1,2,3\}$ occurs infinitely often in $(i_n)_{n\geq 0}$

Example The Tribonacci substitution and its fixed point

• The set of the letter density vectors of AR words has zero measure [Arnoux-Starosta] and even Hausdorff dimension < 2 [Avila-Hubert-Skripchenko]

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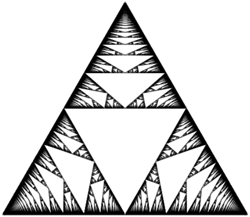

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- The set of the letter density vectors of AR words has zero measure [Arnoux-Starosta] and even Hausdorff dimension < 2 [Avila-Hubert-Skripchenko]
- There exist AR words that do not have bounded symbolic discrepancy [Cassaigne-Ferenczi-Zamboni]
- There exist AR words that are (measure-theoretically) weak mixing [Cassaigne-Ferenczi-Messaoudi]

Example

Let $(i_n) \in \{1,2,3\}^{\mathbb{N}}$ be the fixed point of Tribonacci substitution

$$\sigma\colon 1 \mapsto 12,\ 2 \mapsto 13,\ 3 \mapsto 1$$

$$(i_n) = \sigma^{\infty}(1) = 121312112131212131211213$$

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$$\text{Take} \quad \omega = \lim_{n\to\infty} \sigma_{i_0}\sigma_{i_1}\cdots\sigma_{i_n}(1)$$

We use (i_n) as a directive sequence

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Theorem [B-Steiner-Thuswaldner] (X_ω, Σ) has pure discrete spectrum for any Arnoux-Rauzy word ω whose directive sequence (i_n) belongs to the shift generated by a primitive substitution

S-adic Pisot dynamics

Theorem [B.-Steiner-Thuswaldner]

- For almost every $(\alpha, \beta) \in [0, 1]^2$, the S-adic system provided by the Brun multidimensional continued fraction algorithm applied to (α, β) is measurably conjugate to the translation by (α, β) on the torus $\mathbb{T}^2$
- For almost every Arnoux-Rauzy word, the associated S-adic system has discrete spectrum

Proof Based on

- "adic IFS" (Iterated Function System)
- Theorem [Avila-Delecroix]
  - The Arnoux-Rauzy S-adic system is Pisot
- Theorem [Avila-Hubert-Skripchenko]
  - A measure of maximal entropy for the Rauzy gasket
- Finite products of Brun/Arnoux-Rauzy substitutions have discrete spectrum [B.-Bourdon-Jolivet-Siegel] Finiteness property

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S-adic Pisot conjecture Every unimodular and algebraically irreducible S-adic Pisot system is measure-theoretically conjugate to a Kronecker map

Pisot S-adic systems

- Let $\mathcal{S}$ be a set of unimodular substitutions
- Let (D, Σ, ν) with $D \subset \mathcal{S}^{\mathbb{N}}$ be an ergodic subshift equipped with a probability measure ν. We assume log-integrability
- We consider the generic behaviour of the cocyle $A_n(\boldsymbol{\sigma}) = M_{\sigma_0} \cdots M_{\sigma_n}$ for $\boldsymbol{\sigma} = (\sigma_n) \in D$

The S-adic system (D, S, ν) is said to Pisot S-adic if the Lyapunov exponents $\theta_1, \theta_2, \ldots, \theta_d$ of (D, Σ, ν) satisfy

$$\theta_1 > 0 > \theta_2 \geq \theta_3 \geq \cdots \geq \theta_d$$

The PRICE to pay

$$M_{[k,\ell]} = M_k \cdots M_{\ell-1} \qquad \omega^{(k)} = \lim_{n\to\infty} \sigma_{i_k}\sigma_{i_1}\cdots\sigma_{i_n}(a) \quad \rightsquigarrow X^{(k)}$$

- (P) Primitivity $\forall k,\ M_{[k,\ell)} > 0$ for some $\ell > k$
- (R) Recurrence For each ℓ there exist $n = n(\ell)$ s.t.

$$(\sigma_0, \sigma_1, \ldots, \sigma_{\ell-1}) = (\sigma_n, \sigma_{n_k+1}, \ldots, \sigma_{n+\ell-1})$$

- (I) Algebraic irreducibility for each $k \in \mathbb{N}$, the characteristic polynomial of $M_{[k,\ell)}$ is irreducible for all sufficiently large ℓ
- (C) C-discrepancy There is $C > 0$ such that $n = n(\ell)$ can be chosen such that $X_{\boldsymbol{\sigma}}^{(n+\ell)}$ has discrepancy bounded by C
- (E) Generalized Left Eigenvector

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- (P) Primitivity $\forall k$, $M_{[k,\ell)} > 0$ for some $\ell > k$

  cf. Furstenberg's condition

  There exists $h \in \mathbb{N}$ and a positive matrix B such that $M_{[\ell_k - h, \ell_k)} = B$ for all $k \in \mathbb{N}$
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$$\lim_{k\to\infty} \mathbf{v}^{(n_k)}/\|\mathbf{v}^{(n_k)}\| = \mathbf{v}$$

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If (D, Σ, ν) is a Pisot S-adic shift such that there exists a cylinder of positive measure in D corresponding to a substitution with positive incidence matrix, then the property PRICE holds

The two-letter S-adic Pisot case

Theorem [B.-Minervino-Steiner-Thuswaldner] Let $\boldsymbol{\sigma} = (\sigma_n)_{n\in\mathbb{N}}$ be a primitive and algebraically irreducible sequence of unimodular substitutions over a two-letter alphabet

Assume that there is $C > 0$ such that

- for each $\ell \in \mathbb{N}$, there is $n \geq 1$ with

$$(\sigma_n, \ldots, \sigma_{n+\ell-1}) = (\sigma_0, \ldots, \sigma_{\ell-1}) \quad \text{recurrence}$$

- $X_{\boldsymbol{\sigma}}^{(n+\ell)}$ has C–bounded discrepancy

Then the S-adic shift $X_{\boldsymbol{\sigma}}$ has pure discrete spectrum

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Remarks

- Strong coincidences hold
- Two-letter alphabet $\rightsquigarrow$ algebraic irreducibility and Pisot property

Bratteli-Vershik representation and recognizability

We are given a directive sequence $\boldsymbol{\sigma} = (\sigma_n)$

$$\omega^{(n)} \in \bigcap_k \sigma_n \cdots \sigma_k \{0,1\}^{\mathbb{Z}}, \quad \omega^{(n)} = \sigma_n(\omega^{(n+1)})$$

$$\omega^{(n)} \rightsquigarrow \quad X^{(n)} \rightsquigarrow \quad X = (X^{(n)}, \Sigma)_n$$

Is $\sigma_n : X^{(n)} \to X^{(n+1)}$ a homeomorphism?

Do we have recognizability under the assumption of primitivity?
aperiodicity?

Bratteli-Vershik representation and recognizability

We are given a directive sequence $\boldsymbol{\sigma} = (\sigma_n)$

$$\omega^{(n)} \rightsquigarrow \quad X^{(n)} \rightsquigarrow \quad X = (X^{(n)}, \Sigma)_n$$

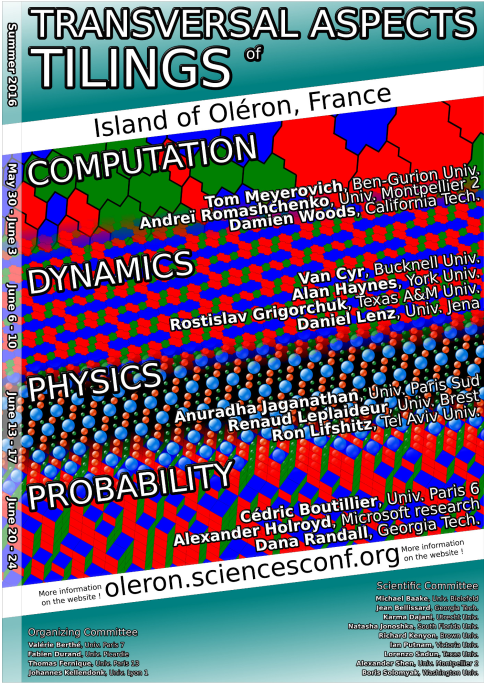
The S-adic shift X is recognizable if for a.e. every $x \in X^{(n)}$, there exists a unique (k, y) such that

$$x = \Sigma^k \sigma_n(y), \text{ with } 0 \leq k < |\sigma_n(y_0^{(n+1)})| \text{ and } y \in X^{(n+1)}$$

Theorem [B.-Steiner-Thuswaldner-Yassawi]

- If (X, Σ) is a recognizable S-adic system, then (X, Σ) is measure-theoretical isomorphic to its associated Bratteli-Vershik model (X_B, φ_ω).
- Property PRICE implies recognizability.

Substitutive case [Mossé,Bezuglyi-Kwiatkowski-Medynets,Vershik-Livshits,Durand-Host-Skau,Canterini-Siegel]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Damien Woods, California Tech.
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