Workshop on Analytic Methods in Classical and Quantum Mechanics
June 27-July 1, 2016

Centro di Ricerca Matematica Ennio De Giorgi
Pisa

TITLES AND ABTRACTS

MINICOURSES

Minicourse 1

Frédéric Faure (Université de Grenoble)

A semi-classical approach for the Ruelle-Pollicott spectrum of hyperbolic dynamics

Abstract: Uniformly hyperbolic dynamics (Anosov, Axiom A) have "sensitivity to initial conditions" and manifest "determinist chaotic behavior", e.g. mixing, statistical properties etc. In the 70', David Ruelle, Rufus Bowen and others have introduced a functional and spectral approach in order to study these dynamics which consists in describing the evolution not
of individual trajectories but of functions and observing the convergence towards equilibrium in the sense of distribution. This approach has progressed and these last years, it has been shown by V. Baladi, C. Liverani, M. Tsujii, N. Roy, J. Sjöstrand, S. Dyatlov, C. Guillarmou, M. Zworski, and others that the generator of this evolution operator ("transfer operator") has a discrete spectrum, called "Ruelle-Pollicott resonances" which describes the effective convergence and fluctuations towards equilibrium and that this spectrum is determined from the periodic orbits (using the Atiyah-Bott trace formula).
Due to hyperbolicity, the chaotic dynamics sends the information towards small scales (high Fourier modes) and technically it is convenient to use "semiclassical analysis" which permits to treat fast oscillating functions. More precisely it is appropriate to consider the dynamics lifted in the cotangent space T*M of the initial manifold M (this is an Hamiltonian flow). We observe that at fixed energy (frequency along the flow direction), this lifted dynamics has a compact non wandering set called the trapped set and that the lifted dynamics on T*M scatters on this trapped set. Then the existence and properties of the discrete Ruelle-Pollicott spectrum follows from the uncertainty principle (i.e. effective discreteness of T*M) and rejoin a more general theory of semiclassical analysis developed in the 80' by B. Helffer and J. Sjöstrand called "quantum scattering on phase space".

The minicourse will explain some aspects of this topic and will have three parts:

1) For general Anosov flows we will explain the existence of an intrinsic discrete spectrum
of Ruelle Pollicott resonances in specific anisotropic Sobolev spaces.

2) For the special case of contact Anosov flow, e.g. geodesic flow, we will explain that the above spectrum is structured into vertical bands.

3) We discuss some consequences of the previous results: relation between the spectrum and periodic orbits and in case of geodesic flow, the relation with the Laplacian operator and quantum chaos. This will suggest many open questions.

Minicourse 2

Viviane Baladi (Institut de Mathématiques de Jussieu)

A brief tour in the jungle of anisotropic Banach spaces for hyperbolic dynamics

Abstract: We consider (mostly discrete-time) C^r hyperbolic dynamics, for r>1. We shall describe several of the available anisotropic spaces used to study the spectra of weighted transfer operators (including applications to Gibbs states and their statistical properties) and the dynamical determinants (or zeta functions).

Minicourse 3

Livio Flaminio (Université Lille I)

Effective Mixing, Effective Equidistribution and Renormalization.

Abstract: It is well known that for Anosov systems the effective equidistribution of stable foliation is equivalent to some effective mixing property (decay of correlation) for the Anosov system. In the first part of the course we shall review the case of classical horocycle flows. For these flows effective equidistribution can be proved by methods of harmonics analysis together with the renormalization methods. For the classical horocycle flow, the natural “renormalization flow” is given by the geodesic flow. Harmonic analysis is used to obtain tame estimates for the solutions of the cohomological equation for the horocycle flow and renormalization to estimate the projections of the currents associated to orbit intervals onto the space of flow-invariant currents.

In the second part of the course we shall shows how to apply these ideas to the case of the stable foliation of partially hyperbolic automorphisms of some nilmanifolds. As result we obtain effective mixing of these partially hyperbolic automorphisms.

Finally we shall consider the generic case of families of flows on Heisenberg nilmanifolds that are not immediately renormalizable by a diffeomorphism of the ambient manifold. Such flows form a bundle on which a recurrent renormalization dynamics can be defined. Using the bundle renormalization dynamics we can recover effective equidistribution for the “generic” Heisenberg nilflow. This approach was inspired by the renormalization of translation flows on compact surfaces, which is given by the Teichmueller flow on the moduli space of holographic differentials.

If time permits we shall show how one can handle the case of flows for which no recurrent renormalization dynamics exists.

TALKS

Mark Demers (Fairfield University)
Exponential decay of correlations for Sinai billiard flows

Abstract: While billiard maps for large classes of dispersing billiards are known to enjoy exponential decay of correlations, the corresponding flows have so far resisted such analysis. We describe recent results, based on the construction of function spaces on which the associated transfer operator has good spectral properties, which provide a description of the spectrum of the generator of the semi-group. This construction, together with a Dolgopyat-type cancellation argument to eliminate certain eigenvalues, proves that the generator has a spectral gap and that the Sinai billiard flow with finite horizon has exponential decay of correlations. This is joint work with V. Baladi and C. Liverani.

Semyon Dyatlov (MIT)
Spectral gaps via additive combinatorics

Abstract: A spectral gap on a noncompact Riemannian manifold is an asymptotic strip free of resonances (poles of the meromorphic continuation of the resolvent of the Laplacian). The existence of such gap implies exponential decay of linear waves, modulo a finite dimensional space; in a related case of Pollicott--Ruelle resonances, a spectral gap gives an exponential remainder in the prime geodesic theorem.

We study spectral gaps in the classical setting of convex co-compact hyperbolic surfaces, where the trapped trajectories form a fractal set of dimension $2\delta + 1$. We obtain a spectral gap when $\delta=1/2$ (as well as for some more general cases). Using a fractal uncertainty principle, we express the size of this gap via an improved bound on the additive energy of the limit set. This improved bound relies on the fractal structure of the limit set, more precisely on its Ahlfors--David regularity, and makes it possible to calculate the size of the gap for a given surface.

Paolo Giulietti (Universidade Federal do Rio Grande do Sul)

Parabolic dynamics and Anisotropic Banach spaces

Abstract: We present recent results, joint work with C. Liverani. We investigate the relation between the distributions appearing in the study of ergodic averages of parabolic flows (e.g. in the work of Flaminio-Forni) and the ones appearing in the study of the statistical properties of hyperbolic dynamical systems (i.e. the eigendistributions of the transfer operator acting on suitable anisotropic Banach spaces). To avoid, as much as possible, technical issues that would cloud the basic idea, we limit ourselves to a simple flow on the torus. Our main result is that, roughly, the growth of ergodic averages of a class of parabolic flows is controlled by the eigenvalues of a suitable transfer operator associated to the renormalizing dynamics. The connection that we illustrate is expected to hold in considerable generality (see http://arxiv.org/abs/1412.7181).

Konstantin Khanin (University of Toronto)
On renormalization and rigidity for a class of nonlinear interval exchange transformations

Jens Marklof (Bristol University)
Universal hitting time statistics for integrable flows

Abstract: The perceived randomness in the time evolution of ``chaotic" dynamical systems can be characterized by universal probabilistic limit laws, which do not depend on the fine features of the individual system. One important example is the Poisson law for the times at which a particle with random initial data hits a small set. This was proved in various settings for dynamical systems with strong mixing properties. The key result of the present study is that, despite the absence of mixing, the hitting times of integrable flows also satisfy universal limit laws which are, however, not Poisson. We describe the limit distributions for ``generic" integrable flows and a natural class of target sets, and illustrate our findings with two examples: the dynamics in central force fields and ellipse billiards. The convergence of the hitting time process follows from a new equidistribution theorem in the space of lattices, which is of independent interest. Its proof exploits Ratner's measure classification theorem for unipotent flows, and extends earlier work of Elkies and McMullen. Joint work with C.P. Dettmann and A. Strombergsson.

Masato Tsujii (Kyushu University)
Exponential mixing for volume-preserving Anosov flows on three dimensional manifolds.

Abstract: We would like to present a results which shows that almost all volume-preserving Anosov flows on three dimensional manifolds are exponentially mixing. The point of the argument is, in comparison with the celebrated result of Dolgopyat, that we state a quantitative condition on joint non-integrability of the stable and unstable foliation in the case where they are not C^1. We begin with discussing a geometric coherent structure on the normal bundle of each (strong) unstable manifold determined by the flow. Then we formulate a quantitative non-integrability condition using this structure.

Corinna Ulcigrai (Bristol University)

Multiple mixing for area preserving flows

Abstract: In joint work with Adam Kanigowski and Joanna Julaga-Prysmus, we consider smooth area preserving flows on surfaces of higher genus (also known as locally Hamiltonian flows). We show that the typical such flow (in the measure theoretical sense) which has several mixing minimal components, is actually mixing of all orders when restricted to each minimal component (thus verifying Rohlin's conjecture that mixing implies mixing of all orders in this context). In this talk we will first recall and explain the basic mechanism of shearing which has proved key in proving mixing of several classes of parabolic dynamical systems.The proof of multiple mixing exploits quantitative shearing estimates (in the form of a variant of the Ratner property recently introduced by Fayad and Kanigowski to prove an analogous result in the genus one case). We will explain how the underlying hyperbolic renormalization dynamics (given by the Teichmueller geodesic flow or Rauzy-Veech induction) is key to provide Diophantine-like conditions under which the result on area preserving flows holds.

Maciej Zworski (University of California-Berkeley**)**

Ruelle zeta function at zero for surfaces of variable curvature

Abstract: For surfaces of constant negative curvature the Selberg trace formula shows that the order of vanishing of the Ruelle zeta function at 0 is given by the absolute value of the Euler characteristic of the surface. Using simple microlocal arguments we prove that this remains true for any negatively curved sufficiently smooth surface. Joint work with S. Dyatlov.