Trimester on Partial Differential Equations and Calculus of Variations

Centro di Ricerca Matematica "Ennio De Giorgi", Pisa
September 1-December 15, 2006

1. Scientific Committee, long-term visitors and participants

Scientific committee: Luigi Ambrosio (SNS, PISA), **Alice Chang** (Princeton), **Gianni Dal Maso** (SISSA, Trieste), **Craig Evans** (Berkeley), **Felix Otto** (IAM, Bonn), **Michael Struwe** (ETH, Zurich).
The scientific committee was appointed in the summer of 2005, and a first round of invitations was sent in the fall of 2005, addressed to long-term visitors.

Long term visitors: Patrick Bernard (CEREMADE), **Yann Brenier** (NICE), **Matt Gursky** (NOTRE DAME), **Gerhard Huisken** (MPI Potsdam), **Richard D.James** (Minnesota), **Ernst Kuwert** (Freiburg), **Chang-Shou Lin** (Taiwan), **Andrea Malchiodi** (SISSA), **Paul Yang** (Princeton), **Michael Ortiz** (Caltech), **Gilles Francfort** (Paris XIII), **Olivier Druet** (ENS Lyon).

Other participants: S. Brendle (Princeton University), F. Merle (Université de Cergy-Pontoise), F. Planchon (Université Paris 13), R. Mueller (ETH, Zurich), M. Dafermos (Cambridge University), C. Mantegazza (SNS, Pisa), O. Schnuerer (Berlin University), P. Topping (Warwick University), J. Viaclovsky (MIT), G. Friesecke (Munich University), A. De Simone (SISSA), X. Blanc (Paris VI), A. Chambolle (Ecole Polytechnique), S. Conti (Duisburg), A. Braides (Roma II), S. Müller (MPI, Leipzig), A. Garroni (Roma I), F. Tomarelli (Politecnico di Milano), G. Del Piero (Ferrara), I. Fragalà (Politecnico Milano), C. Le Bris (CERMICS), I. Fonseca (Carnegie Mellon University), K.T. Sturm (Bonn), W. Gangbo (Georgia Tech.), L. De Pascale (Pisa), A. Fathi (ENS Lyon), S. Fang (Bourgogne), G. Bouchitté (Toulon), A. Figalli (SNS), G. Buttazzo (Pisa), B. Piccoli (IAC, Rome), G. Carlier (CEREMADE), R. McCann (Toronto), G. Savaré (Pavia), A. Pratelli (Pavia), C. De Lellis (Zurich), S. Serfaty (Courant, New York).

Students with fellowships: Michael Eichmar (Stanford), Antoine Gloria (Paris), Kim Hwakil (Georgia Tech), Ryan Hynd (Berkeley), Cheikh Birahim Ndiaye (SISSA), Laurent Nocquet Jeol (Paris Dauphine), Maria Rosaria Amendola (Salerno), Marco Barchiesi (Sissa), Alice Fiaschi (Sissa), Luca Martinazzi (ETH), Sandra Rita Martinez (Buenos Aires), Giampiero Palatucci (Roma 3), Marc Oliver Rieger (ETH), Helia Serrano (Universidad Castilla La Mancha), Nigel Wilkin-Smith (Australian National University), Ni Xiang (Nanjing University), Caterina Ida Zeppieri (Roma I), Meiyue Jiang (Beijing).

2. Main Activities

2.1. Meetings

- "Nonlinear evolution problems" September 19 to 23.
- "Variational methods in Material Science" October 10 to 14.
- "Optimal transport: theory and applications" November 14 to 18.

2.2. Lecture series

Lectures on Hamiltonian and Geometric PDE's

Yann Brenier: Relative entropy methods for singular perturbations of hamiltonian PDE's.

Patrick Bernard: Optimal transportation and Mather theory.

Michael Struwe: Quantization issues in Geometric PDE's.

Lectures on multiscale modeling of materials
M. Ortiz, Caltech, USA

(1) From subgrain mocrostructures to polycrystals.
(2) From atoms to dislocation structures.
(3) From quantum mechanics to lattice defects.

Lectures on Geometric Analysis

Gerhard Huisken: Isoperimetric inequalities and the concept of mass in General Relativity.

Ernst Kuwert: Analysis for the Willmore functional.

Andrea Malchiodi: High-order curvatures in conformal geometry.

2.3. Seminars

P.Yang: "Isoperimetric inequality in pseudohermitian geometry"

Emmanuel Hebey: "Compactness for critical elliptic systems in potential form"

Nonlinear evolution problems

PROGRAM

September 19.

14.30-15.00. Opening address.
15.00-15.55. A. Chang. (Princeton University) Boundary regularity of a 4th order elliptic system.
16.00-16.55. S. Brendle. (Princeton University) Boundary regularity of minimal maps between surfaces.
17.00-17.30. Coffee break.
17.30-18.25. M. Gursky. (Notre Dame University) A non-local flow for Riemann surfaces.

September 20.

9.30-10.25. G. Huisken. (MPI Institut, Potsdam) Mean curvature type flows and an isoperimetric concept for mass and quasilocal mass in General Relativity.
10.30-11.00. Coffee break.
11.00-11.55. E. Kuwert. (Freiburg University) Conformal surfaces with bounds on their Willmore energy.

15.00-15.55. J. Viaclovsky. (MIT) Fully nonlinear equations in conformal geometry.
16.00-16.55. C.S. Lin. (National Chung-Cheng University, Taiwan) Green functions, Elliptic functions and the Mean Field equations.
17.00-17.30. Coffee break.
17.30-18.25. O. Druet. (ENS Lyon) Quantification of blow-up levels for a 2-d elliptic equation with critical exponential nonlinearity.

September 21.

9.30-10.25. F. Merle. (Université de Cergy-Pontoise) On global existence for solutions to the focusing energy-critical NLS.
10.30-11.00. Coffee break.
11.00-11.55. A. Malchiodi. (SISSA, Trieste) Q-curvature flow on $\mathbf{S}^4$.
Free afternoon.

September 22.

9.30-10.25. F. Planchon. (Université Paris 13) Global existence for H^1 critical waves in $3d$ domains.
10.25-11.00. Coffee break.
11.00-11.55. R. Mueller. (ETH, Zurich) Differential Harnack inequalities and monotone quantities.

15.00-15.55. M. Dafermos. (Cambridge University) The problem of stability for black hole spacetimes in general relativity.

16.00-16.55. C. Mantegazza. (SNS, Pisa) Some remarks on Ricci solitons.
17.00-17.30. Coffee break.
17.30-18.25. O. Schnuerer. (Berlin University) Surfaces contracting with speed $|A|^2$.

September 23.

9.30-10.25. P. Topping. (Warwick University) Ricci flow, entropy and optimal transportation.
10.30-11.00. Coffee break.
11.00-11.55. M. Struwe. (ETH, Zurich) Partial regularity for harmonic maps, revisited.

ABSTRACTS

Simon Brendle. A minimal map is an area preserving diffeomorphism whose graph is a minimal surface. We will discuss the regularity of such maps near the boundary. To this end, we need to control the tangent plane to the minimal surface at the boundary.

Alice Chang. In this talk, I will report some recent joint work of Sophie Chen, Paul Yang and myself on a regularity problem of Bach flat metrics on 4-manifolds with boundary. I will discuss the set up, the ϵ regularity and a compactness result for the solution of the problem.

Olivier Druet. We consider sequences of solutions of some 2d elliptic with critical Moser-Trudinger growth and we prove that the blow-up can occur only in some standard way. We provide a quantification of blow-up levels for such sequences. I will emphasize on the similarities and differences with the Yamabe type equations in higher dimensions.

Matthew Gursky. A non-local flow is defined for compact Riemann surfaces. The metric evolution consists of two terms, the (normalized) Ricci flow and a non-local term, and is coupled to an elliptic equation. Solvability of the elliptic equation is related to conformal Killing vector fields, and a convergence result it proved.

Ernst Kuwert. The Willmore integral of a closed surface $\Sigma \subset \mathbf{R}^n$ is the integral

$$\mathcal{W} = \frac{1}{4} \int_{\Sigma} |H|^2 \, d\mu_g \ .$$

This integral is invariant under Möbius transformations of $\mathbf{R}^n$. We show that for a torus in $\mathbf{R}^3$ with $\mathcal{W}(\Sigma) \leq 8\pi - \delta$, the induced metric is uniformly equivalent to a flat metric with constant depending only on δ. The results are joint work with Reiner Schätzle (Universität Tübingen).

Chang-Shou Lin. In this talk, two main results are presented. One is to apply the elliptic functions to solve a mean field equations on torus, and apply these results of mean field equations to prove Green functions of any torus have at most five critical points.

Andrea Malchiodi. We consider the problem of prescribing the Q-curvature on the standard sphere S4 through a conformal change of metric. The problem is not always solvable because of some Kazdan-Warner obtructions, and some "topological" conditions on the function to be prescribed have to be imposed. We study the problem though a parabolic flow and reduce it to a finite-dimensional one via asymptotic analysis. Stationary points are then found using Morse theory. This is joint work with M.Struwe.

Carlo Mantegazza. I will discuss some classification results for Ricci solitons, that is, self-similar solutions of the Ricci Flow. New simpler proofs of some known results will be presented. In detail, we will take the equation point of view, avoiding to use the tools provided by considering the dynamical properties of the Ricci flow.

Frank Merle. We give a sharp criterion for global existence for solution to the focusing energy- critical NLS.

Reto Müller. We will present some of the relations between Li-Yau type Harnack inequalities and monotone integral quantities, such as Perelman's entropy functionals or his L-functional. We will also present some analogous facts for the heat equation on a static manifold, based on an entropy functional found by Lei Ni.

Fabrice Planchon. We prove that the defocusing quintic wave equation, with Dirichlet boundary conditions, is globally well posed on $H_0^1(\Omega) \times L^2(\Omega)$ for any smooth (compact) domain $\Omega \subset \mathbf{R}^3$. The main ingredient in the proof is an L^5 spectral projector estimate, obtained recently by Smith and Sogge, combined with a detailed study of the boundary value problem.

Oliver Schnuerer. We consider strictly convex surfaces embedded in $\mathbf{R^3}$ that contract with normal velocity equal to $|A|^2$, the square of the norm of the second fundamental form,

$$\dot{X} = -|A|^2 \nu.$$

We obtain convergence to a point in finite time. After appropriate rescaling, these surfaces converge smoothly to round spheres. The key observation is that

$$\max_{M_t} \frac{(\lambda_1 + \lambda_2)(\lambda_1 - \lambda_2)^2}{\lambda_1 \lambda_2}$$

is monotone in time under this flow, where λ_i are the principal curvatures of the surfaces M_t. We describe the algorithm that we used to find this test function.

Peter Topping. I will show how Ricci flow provides the natural setting for a number of results from Geometry and Analysis. One part of the talk will be joint work with Robert McCann.

Jeff Viaclovsky. I will discuss local Holder and $W^{1,p}$ estimates for solutions of some fully nonlinear equations in conformal geometry, and analyze the behavior of singular solutions in punctured balls. I will then show how these estimates are used in the solution of the σ_k-Yamabe problem for $k > n/2$. This is joint work with M. Gursky.

Variational Methods in Material Science

PROGRAM

October 10.

15.00-15.10. Opening address.
15.10-16.05. R.D. James (University of Minnesota) New observations on the origins of hysteresis in phase transformations and metastability in the calculus of variations.
16.10-17.05. G. Francfort (Paris XIII) Variational fracture and minimality.
17.05-17.30. Coffee break.
17.30-18.25. G. Friesecke (Munich University) Minimum energy configurations of classical charges.

October 11.

9.30-10.25. A. De Simone (SISSA) Liquid crystals elastomers: modeling, analysis, and simulation.
10.30-11.00. Coffee break.
11.00-11.55. X. Blanc (Paris VI) Stochastic lattices and their macroscopic limits.
15.30-16.25. G. Dal Maso (SISSA) Quasistatic evolution problems in plasticity with softening.
16.30-17.00. Coffee break.
17.00-17.55. A. Chambolle (Ecole Polytechnique) Rigid displacements with cracks.

October 12.

9.00-9.55. S. Conti (Duisburg) A sharp-interface limit for the singularly perturbed Eikonal functional.
10.00-10.55. A. Braides (Roma II) Asymptotic analysis of binary discrete systems.
11.00-11.30. Coffee break.
11.30-12.25. F. Otto (IAM, Bonn) A compactness result in micromagnetics.
Free afternoon.

October 13.

9.00-9.55. S. Müller (MPI, Leipzig) Are the Von Karman equations a good model for buckling ?
10.00-10.55. A. Garroni (Roma I) Variational methods for plasticity by homogenization of discrete dislocations.
11.00-11.30. Coffee break.
11.30-12.25. F. Tomarelli (Politecnico di Milano) A variational principle for plastic hinges.
15.30-16.25. G. Del Piero (Ferrara) Regularized descriptions of fracture.
16.30-17.00. Coffee break.
17.00-17.55. I. Fragalà (Politecnico di Milano) The compliance of thin plates: modelling and optimization.

October 14.

9.30-10.25. `C. Le Bris` (CERMICS) Existence and uniqueness of solutions to Fokker-Planck type equations with irregular coefficients.
10.30-11.00. Coffee break.
11.00-11.55. `I. Fonseca` (Carnegie Mellon University) Surfactants in foam stability: a phase field model.

ABSTRACTS

Xavier Blanc.
We will present a joint work with C. Le Bris and P.-L. Lions dealing with the definition of the energy of infinite set of atoms. Starting from the very simple case of a periodic lattice with classical energy, we will extend this kind of results to stochastic lattices and/or quantum models of Thomas-Fermi type. In a second step, we will make the link between these kind of energies and macroscopic hyperelastic energies.

Andrea Braides.
The discrete-to-continuous analysis of binary discrete systems already contains many of the features and problems of more complex interactions. We show how a multi-scale analysis of these systems exhibits bulk energies of different type, phase and anti-phase boundaries, multi-phase limits, and gives way to the description of vortex singularities in the vector case. It also allows a simple introduction of a random dependence on the lattice, whose continuous counterpart is easily described in the "dilute case", while it becomes more delicate in the general case. Work in collaboration with Alicandro, Cicalese and Piatnitski.

Antonin Chambolle.
We will discuss some properties of (G)SBV functions. In general, the curl of the approximate gradient of such a function can apparently be a nasty object. However, in a simple case (bounded gradient, jump with finite surface), it is shown to be a measure controlled by the surface measure of the jump. A consequence is a "piecewise rigidity" result for displacements with gradient almost everywhere a rotation, and jump with finite surface. This is a joint work with A. Giacomini and M. Ponsiglione.

Sergio Conti. (joint work with Camillo De Lellis)
The singularly perturbed Eikonal functional in two dimensions arises in the modeling of different physical problems, including liquid crystals blistering in thin films, convection patterns, and magnetism in thin films. From a mathematical viewpoint, it presents several difficulties arising from the interplay of the gradient structure with a connected, nonconvex zero-set of the leading order energy term. We present a construction for an optimal recovery sequence for all limit functions u such that $\nabla u \in BV(\Omega; \mathbf{S}^1)$. Together with previous work by several other authors, this permits to prove Γ-convergence for the same set of limiting functions.

Gianni Dal Maso.

In plasticity theory the term softening refers to the reduction of the yield stress as plastic deformation proceeds. We deal with this problem in the quasistatic case, in the framework of small strain associative elastoplasticity. The presence of a nonconvex term due to the softening phenomenon requires the extension of a variational framework proposed by Mielke to the case of a nonconvex energy functional. In this problem the use of global minimizers in the corresponding incremental problems is not justified from the mechanical point of view. We analize a different selection criterion for the solutions of the quasistatic evolution problem, based on a viscous approximation. In view of the nonconvexity of the problem, taking the limit as the artificial viscosity parameter tends to zero leads to a weak formulation of the problem in a space of Young measures. Moreover, since the growth exponent of the energy is one, we need a suitable notion of generalized Young measure in order to deal with concentration effects. Finally, the classical notion of total variation of a time-dependent function on a time interval has to be extended to time-dependent families of Young measures. This enables us to define, in this generalized context, a notion of dissipation, which plays a crucial role in Mielke's variational approach. Some examples show that smooth initial data may lead, after a critical time, to a Young measure solution with concentration phenomena.

Antonio De Simone.
Liquid crystal elastomers (LCEs) are rubbery solids exhibiting many of the partially ordered mesophases (mematc, smectic A, smectic C) of liquid crystals. The resulting phase transitions are at the root of their many interesting material properties, such as soft elasticity and shape memory effects.
In recent years, the application of modern tools in the calculus of variations (in particular, relaxation of nonconvex energy functionals) has contributed significantly to the understanding of the material properties of LCEs. We will review some of the results justifying this claim, which have been obtained in joint papers with J. Adams, S. Conti, and G. Dolzmann.

Irene Fonseca.
The role of surfactants in stabilizing the formation of bubbles in foams is studied using a phase-field model The analysis is centered on a van der Walls-Cahn-Hilliard-type energy with an added term accounting for the interplay between the presence of a surfactant density and the creation of interfaces. In particular, it is concluded that the surfactant segregates to the interfaces, and that the prescription of the distribution of surfactant will dictate the locus of interfaces, what is in agreement with experimentation.
This is joint work with Massimiliano Morini and Valeriy Slastikov.

Gilles Francfort.
The variational theory of brittle fracture, which has developed in the past few years, is based on the time evolution of global minimizers of the energy functionals. In this talk I will briefly recall the main results, then concentrate on the issues brought about by global minimality and focus on possible alternatives. There, the theory is far from complete, especially in dimensions greater than one.

Ilaria Fragalà.

We consider an optimal design problem, which consists in finding the optimal distribution of a prescribed amount of plate-like material in a certain design region, in order to minimize the compliance under a given system of forces. We identify admissible mass distributions to positive measures with prescribed integral mean, thus allowing both diffused and concentrated solutions. By this way, we immediately get the existence of an optimal design, and the minimal compliance can be recovered simply by maximizing a linear form under an Hessian constraint.
In the first part of the talk I will show how this model can be derived from 3D-linear elasticity, by discussing the asymptotic behaviour of a sequence of optimal elastic compliance problems, in the double limit when both the maximal height of the design region and the total volume of the material tend to zero.
In the second part I will give necessary and sufficient optimality conditions, which can be used in order to compute the minimal value of the compliance and to determine analytically some optimal plates.
The results are contained in recent joint works with G. Bouchitté.

Gero Friesecke.
We study minimum energy configurations of N particles in $\mathbf{R}^3$ of charge -1 ('electrons') in the potential of M particles of charges $Z_\alpha > 0$ ("atomic nuclei"). In a suitable large-N limit, we determine the asymptotic electron distribution explicitly, showing in particular that the number of electrons surrounding each nucleus is asymptotic to the nuclear charge ("screening"). The proof proceeds by establishing, via Gamma-convergence, a coarse-grained variational principle for the limit distribution, which can be solved explicitly. (Joint work with Stephane Capet.) As an application, we give a simple proof of the celebrated Dyson-Lenard theorem in quantum mechanics.

Adriana Garroni.
Dislocations are topological defects in crystal that are considered responsible for plastic deformations. We consider a 2D model for edge dislocations, where the deformation has a singularity on points that represent dislocations, while the crystal behaves elastically far from the core. This model is very close to the 2D Ginburg-Landau model for the study of vortices in superconductors. We study, in a dilute regime, the limit as the number of points (dislocations) tends to infinity and we obtain as limit problem an elasto-plastic model, given by the elastic energy and a term depending on the Curl of the plastic deformation (the dislocations density).

Richard D. James.
We present some recent measurements of hysteresis that resulted from a systematic program of tuning of the lattice parameters of some alloys by changing composition. The lattice parameters were tuned so that a certain nongeneric condition of compatibility was satisfied. It was observed that there is a sharp drop of the hysteresis of the transformation at the special lattice parameters. The data has some fascinating features, including an apparent singularity. Some exploratory calculations suggest that this is related to metastability phenomena in the calculus of variations, suggesting a sensitive dependence of local minimizers on certain conditions of compatibility. Joint work with Jerry Zhang and Stefan Müller.

Claude Le Bris.
We study the existence and the uniqueness of the solution to a class of Fokker-Planck type equations with irregular coefficients, more precisely with coefficients in Sobolev spaces $W^{1,p}$. Our arguments are based upon the DiPerna-Lions theory of renormalized solutions to linear transport equations and related equations. The work extends the results of our previous work, where only the simpler case of a Fokker-Planck equation with constant diffusion matrix was addressed. The consequences of the present results on the well-posedness of the associated stochastic differential equations are also outlined.

OPTIMAL TRANSPORT: THEORY AND APPLICATIONS

PROGRAM

November 14.

15.00-15.10. Opening address.
15.10-16.05. K.T. Sturm (Bonn) Optimal Transportation, Ricci Curvature and Diffusions on the L^2-Wasserstein Space.
16.10-17.05. W. Gangbo (Georgia Tech.) Euler–Poisson systems as energy minimizing paths in the Wasserstein space $\mathcal{P}_2(\mathbf{R})$.
17.05-17.30. Coffee break.
17.30-18.25. L. De Pascale (Pisa) Optimal transportation of certain distributions and applications.

November 15.

9.30-10.25. A. Fathi (ENS Lyon) Optimal transport on non-compact manifolds.
10.30-11.00. Coffee break.
11.00-11.55. S. Fang (Bourgogne) Transportation cost inequalities and optimal transport maps on loop groups.

15.00-15.55. G. Bouchitté (Toulon) Dimension reduction for linear constraint problems. Applications in optimal design.
16.00-16.55. A. Figalli (SNS) Ricci curvature and displacement convexity.
17.00-17.30. Coffee break.
17.30-18.25. Y. Brenier (Nice) Monge-Kantorovich stability and L^2 formulation of multidimensional scalar conservation laws.

November 16.

9.00-9.55. G. Buttazzo (Pisa) Asymptotics for a compliance-location problem.
10.00-10.55. B. Piccoli (IAC, Rome) Traffic flow on networks.
11.00-11.30. Coffee break.
11.30-12.25. G. Carlier (CEREMADE) Optimal transportation with traffic congestion and Wardrop equilibria.
Free afternoon.

November 17.

9.00-9.55. L. Ambrosio (SNS) Variational models for incompressible Euler equations and measures on action-minimizing paths.
10.00-10.55. R. McCann (Toronto) Free Boundaries in Optimal Transport.
11.00-11.30. Coffee break.
11.30-12.25. G. Savaré (Pavia) Gradient flows in metric/Wasserstein spaces under lower curvature bounds.

15.30-16.25. `A. Pratelli` (Pavia) On the sufficiency of c-cyclical monotonicity for the optimality of transport plans.
16.30-17.00. Coffee break.
17.00-17.55. `C. De Lellis` (Zurich) Estimates for ordinary differential equations with Sobolev coefficients.

November 18.

9.30-10.25. `S. Serfaty` (Courant, New York) A gradient flow approach to an evolution problem arising in superconductivity.
10.30-11.00. Coffee break.
11.00-11.55. `P. Bernard` (CEREMADE) Young measures, superposition and transport.

ABSTRACTS

Luigi Ambrosio. *Variational models for incompressible Euler equations and measures on action-minimizing paths.* I will describe some work in progress with A.Figalli on the variational models for incompressible Euler equations introduced and studied by Brenier in a series of papers from 1989 to 1999. We show that the purely Lagrangian model introduced in the 1989 paper is equivalent to the mixed Eulerian-Lagrangian one of 1999. As a consequence, many results of the two papers on the existence and regularity of the pressure field P can be combined. By introducing more general first variations for this problem, we also investigate necessary and sufficient minimality conditions at the level of single fluid paths, proving a BV regularity of the velocity of a typical path, and a kind of minimality property with respect to the (nonsmooth) Lagrangian

$$L(t,x,p) := \frac{1}{2}|p|^2 + P(t,x)$$

This provides one more link between optimal transportation and the theory of action-minimizing measures. In the end of the lecture I will illustrate some of the (many) open problems.

Guy Bouchitté. *Dimension reduction for linear constraint problems. Applications in optimal design.* It is a joint work with Ilaria Fragalà motivated by the dual formulation of the so called Michell's problem where it appears, like in Monge-Kantorovich theory, a linear problem with constraint on the first order gradient. The Hessian counterpart of this has been recently introduced and studied in [1].
The goal of this lecture is to show how the aboved mentioned Hessian constrained problem can derived through a 3D-2D reduction analysis and how it arises naturally in optimal design problems for thin plates.
To that aim, we study the asymptotic behaviour of a sequence of optimal elastic compliance problems, in the double limit when both the maximal height of the design region and the

total volume of the material tend to zero. In the vanishing volume limit, a sequence of linear constrained vector first order problem is obtained [2], which in turn - in the vanishing thickness limit - produces a new linear constrained problem where both first and second order gradients appear. When the load is suitably chosen, only the Hessian constraint is active, and we recover exactly the plate optimization problem studied in [1]. Some attention is also paid to the possible different approaches to the afore mentioned double limit process, in both the cases of ficticious and non-ficticious materials, which might favour some debate on the modelling of thin plates.

[1] G . Bouchitté and I. Fragalà: Optimality conditions for mass design problems and applications to thin plates. *Arch. Rat. Mech. Analysis*, to appear.

[2] G. Bouchitté: Optimization of light structures: the vanishing mass conjecture. *Homogenization, 2001 (Naples).* GAKUTO Internat. Ser. Math. Sci. Appl. **18** Gakkötosho, Tokyo (2003), 131-145.

Giuseppe Buttazzo. *Asymptotics for a compliance-location problem.* The characterization of asymptotical configurations in the optimal location of resources recently received a lot of attention, and the approach based on $\Gamma-$convergence seems very fruitful. We quote for instance

- [1] where the so-called *location problem* of n points is studied in the framework of mass transportation theory.
- [3] where the location problem is studied in dimension two, and it is shown that for large n the optimal configuration approaches the one given by the centers of regular exagons.
- [4] where the same analysis is made for the so-called *irrigation problem*, i.e. with points replaced by connected one-dimensional sets of total length L.

We studied in [2] a similar problem where the optimal location of a given number of point for the elastic compliance functional is considered.

References

[1] G. Bouchitté, C. Jimenez, M. Rajesh: *Asymptotique d'un problème de positionnement optimal.* C. R. Acad. Sci. Paris Ser. I, **335** (2002) 1–6.

[2] G. Buttazzo, F. Santambrogio, N. Varchon: *Asymptotics of an optimal compliance-location problem.* ESAIM-COCV (to appear).

[3] F. Morgan, R. Bolton: *Hexagonal Economic Regions Solve the Location Problem.* Amer. Math. Monthly, **109**, no. 2 (2002), 165–172.

[4] S. Mosconi, P. Tilli: $\Gamma-$*Convergence for the Irrigation Problem.* J. Convex Anal., **12**, no.1 (2005), 145–158.

Yann Brenier. *Monge-Kantorovich stability and L^2 formulation of multidimensional scalar conservation laws.* It has been observed in the last years that one dimensional scalar conservation laws enjoy some contraction properties in Monge-Kantorovich (Wasserstein) distances. (See recent papers by Bolley-Brenier-Loeper and Carrillo-Di Francesco-Lattanzio.) We show that this is a general feature of multidimensional scalar conservation laws and introduce a related L2 formulation that completely recover Kruzhkov's entropy solutions.

Guillaume Carlier. *Optimal transportation with traffic congestion and Wardrop equilibria (joint with C. Jimenez and F. Santambrogio).* In the classical Monge-Kantorovich problem, the transportation cost only depends on the amount of mass sent from sources to destinations and not on the paths followed by this mass. Thus, it does not allow for congestion effects. Using the notion of traffic intensity, we propose a variant taking into account congestion. This leads to an optimization problem posed on a set of probability measures on a suitable paths space. We establish existence of minimizers and give a characterization. As an application, we obtain existence and variational characterization of equilibria of Wardrop type in a continuous space setting.

Camillo De Lellis. *Estimates for ordinary differential equations with Sobolev coefficients.* Inspired by a result by Ambrosio, Lecumberry and Maniglia, in a joint work with Gianluca Crippa we show a simple derivation of integral logarithmic bounds for solutions of ordinary differential equations

$$\begin{cases} \dfrac{d\Phi}{dt}(t,x) = b(t,\Phi(t,x)) \\ \Phi(0,x) = x. \end{cases}$$

These bounds depend only on the L^∞ and $W^{1,p}$ norm of b, and on the compressibility constant of Φ (which in turn can be bound by $\|\operatorname{div} b\|_\infty$). These a-priori estimates allow to recover in a simple way many old and new results about existence, uniqueness, stability and differentiability properties of solutions of ODE's with Sobolev coefficients. As new corollaries, we conclude that the Cauchy problem for transport equation with Sobolev coefficients preserve a mild regularity property of the initial data and we give an affirmative answer to the L^p version of a Conjecture of Bressan.

L. De Pascale. *Optimal transportation of certain distributions and applications.* I will introduce the Monge-Kantorovich problem for certain distributions in a convex, bounded open subset of $\mathbf{R}^n$ and I will discuss some possible application. I will describe a theory of optimal plans, optimal transport density and a problem related to the regularity of optimal plans.

Shizan Fang. *Transportation cost inequalities and optimal transport maps on loop groups.* We shall focus on the construction of optimal transport maps on loop groups.

Albert Fathi. *Optimal transport on non-compact manifolds.* In this joint work with Alessio Figalli, we show how to adapt previous arguments for the Monge transportation problem to the case of non-compact manifolds. in particular when the cost is the square of a Riemannian metric we show the existence of a transport without any further condition on the Riemannian metric (curvature, growth at infinity, or other) besides its completness.

Alessio Figalli. *Ricci curvature and displacement convexity.* Recent research activity by Lott, Villani, Sturm, Von Renesse has been devoted to study the geometry of Riemannian manifolds and, more in general, length spaces, through the geometry of their associated Wasserstein space of probability measures. In particular, it was found that lower bounds on the Ricci curvature tensor can be recast in terms of convexity properties of certain nonlinear functionals defined on spaces of probability measures. I will first recall the notion of measured length space having Ricci curvature bounded from below, using optimal transport and displacement convexity. Then, I will present a recent result obtained in collaboration with C. Villani where we solve a natural problem in this field by establishing the equivalence of several formulations of convexity properties.

Wilfrid Gangbo. *Euler–Poisson systems as energy minimizing paths in the Wasserstein space* $\mathcal{P}_2(\mathbf{R})$. I will describe a variational approach to establish existence and uniqueness of solutions (σ_t, v_t) of the 1–d Euler-Poisson system, mimimizing an action. Here, σ_0, σ_T are prescribed and the time interval $[0, T]$ satisfies $T < \pi$. This extends the concept of the Euler-Poisson system to general measures. These solutions conserve the Hamiltonian, Legendre transform of the Lagrangian appearing in the action. They yield a path $t \to \sigma_t$ on $\mathcal{P}$, the set of probability measures on the real line, and turn out to be characteristic of an infinite-dimensional Hamilton-Jacobi equation on $\mathcal{P}$. The associated Hamiltonian system is nothing but the 1–d Vlasov-Poisson system. When $\sigma_t = \delta_{y(t)}$ is a dirac mass, the Euler-Poisson system reduces to $\ddot{y} + y = 0$, the Hamilton-Jacobi equation is merely finite dimensional and it is given by $\partial_t u + 1/2(\partial_y u)^2 + 1/2y^2 + 1/24 = 0$. (This is a joint work with T. Nguyen and A. Tudorascu).

Robert McCann. *Free Boundaries in Optimal Transport.* Given a distribution $f(x)$ of iron mines throughout the countryside, and a distribution $g(y)$ of factories which require iron ore, the optimal transportation problem of Monge and Kantorovich asks for the mines to be paired with the factories so as to minimize the average (say) Euclidean distance squared between factory and mine. This problem is deeply connected to geometry, inequalities, and nonlinear differential equations, with applications ranging from shape recognition to weather prediction. In the talk I discuss what happens when the production capacity of the mines need not agree with the demand of the factories, so one ships only a certain fraction of the ore being produced, again choosing the locations of factories and mines which remain active so as to minimize total transportation costs. If the mines are continuously distributed in Euclidean space, and positively separated from the factories, the solution will be unique,and is given by pair of domains U and V in $\mathbf{R}^n$, with U containing the active mines and V the active factories. These domains are characterized as the non-contact regions in a double obstacle problemfor the Monge-Ampère equation. We go on to specify conditionson f and g which are sufficient to ensure that U and V have continuously differentiable free boundaries, and that the correspondences: $U \to V$ mapping mines to factories is an homeomorphism or smoother, Hölder continuous up to the free (and part of the fixed) boundary. The results represent joint work with Luis Caffarelli, Preprint #26 at www.math.toronto.edu/mccann

Benedetto Piccoli. *Traffic flow on networks.* We review some recent results on traffic flow on networks using conservation laws models.

Aldo Pratelli. *On the sufficiency of c-cyclical monotonicity for the optimality of transport plans.* The role of the cyclical monotonicity in the study of transport problems is widely known, and it is also well known that this monotonicity is necessary for a transport plan to be optimal, as well as that the sufficiency also holds in many situations. We will discuss the sufficiency issue for general cost functions, giving some results and counterexamples.

Giuseppe Savaré. *Gradient flows in metric/Wasserstein spaces under lower curvature bounds.* We will discuss generation results for gradient flows in metric spaces underlower curvature bounds and their applications to Wasserstein spaces when the quadratic costis induced by a Riemannian distance.

Sylvia Serfaty. *A gradient flow approach to an evolution problem arising in superconductivity.* In a joint work with Luigi Ambrosio, we study an evolution problem arising in superconductivity and describing the transport of vorticity along the gradient of the potential it generates, a sort of dissipative version of Euler's equation. This equation turns out to be a gradient-flow for the Wasserstein metric on probability measures. We use the framework of Ambrosio-Gigli-Savaré to provide results of existence, sometimes uniqueness, and energy-dissipation relations.

Karl-Theodor Sturm. *Optimal Transportation, Ricci Curvature and Diffusions on the L^2-Wasserstein Space.* We introduce and analyze generalized Ricci curvature bounds for metric measure spaces (M, d, m), based on convexity properties of the relative entropy $Ent(\cdot|m)$. For Riemannian manifolds, $Curv(M, d, m) \geq K$ if and only if $Ric_M \geq K$ on M; for the Wiener space, $Curv(M, d, m) = 1$.
One of the main results is that these lower curvature bounds are stable under (e.g. measured Gromov-Hausdorff) convergence. This solves one of the basic problems in this field, open for many years.

Furthermore, we introduce a (more restrictive) curvature-dimension condition $\mathrm{CD}(K, N)$ which implies sharp versions of the Brunn-Minkowski inequality, of the Bishop-Gromov volume comparison theorem and of the Bonnet-Myers theorem. Moreover, it allows to construct a canonical Dirichlet form with Gaussian bounds for the corresponding heat kernel.

Finally, we indicate how to construct a canonical reversible process on the L^2-Wasserstein space of probability measures $\mathcal{P}(\mathbf{R})$, regarded as an infinite dimensional Riemannian manifold. This process has an invariant measure $\mathbf{P}^\beta$ which may be characterized as the "uniform distribution" on $\mathcal{P}(\mathbf{R})$ with weight function $Z^{-1} \exp(-\beta \cdot Ent(\cdot|m))$, where m denotes a given finite measure on $\mathbf{R}$. One of the key results is the quasi-invariance of this measure $\mathbf{P}^\beta$ under push forwards $\mu \mapsto h_* \mu$ by means of smooth diffeomorphisms h of $\mathbf{R}$.