ABSTRACTS

Sub-Riemannian Geometry and Beyond, III

June 19-23, 2023

Mini courses

Luca Capogna

Carnot-Caratheodory visual boundaries

Abstract:
We will provide a very succinct introduction to Gromov hyperbolic spaces, and to the quasiconformal geometry of their visual boundaries. After looking at some basic examples, real and complex hyperbolic spaces, and more general noncompact negatively curved symmetric spaces, we will focus on the work of Balogh and Bonk, who proved that strictly pseudoconvex domains endowed with their Bergman metric are Gromov hyperbolic.

Pierre Pansu

Nilpotent groups and their asymptotic cones.

Abstract:
The contents of this mini course are:
- Asymptotic cones of nilpotent groups. Speed of convergence. Application to quasi-isometries.
- Carnot groups as spaces with dilations. Proof of the characterization theorem.

Yves Cornulier

Introduction to the geometry of Lie groups

Abstract:
I will present geometry of Lie groups, with a view oriented towards quasi-isometry classification. This will include general basics on large-scale geometry of metric spaces. I will then notably focus on the class of groups of polynomial growth.

Roman Sauer

Cohomological quasi-isometry invariants

Abstract:
I will explain Gromov's coupling point of view of quasi-isometry. It turns out that this notion is better suited to cohomology. After discussing the QI-invariance of finiteness conditions, cohomological dimension, the focus will be on the invariance of the cohomology algebra of nilpotent groups.

Conference talks

Wednesday June 21, 2023

Frédéric Paulin

Counting and equidistribution in quaternionic Heisenberg groups

Abstract:
We prove an effective counting and equidistribution theorem for rational points in quaternionic Heisenberg groups endowed with their Carnot-Carathéodory distances. The proofs use methods and results from quaternionic hyperbolic geometry (in particular its geometry at infinity), arithmetic groups and ergodic theory of the geodesic flow in negatively curved spaces. This is a joint work with Jouni Parkkonen.

Tullia Dymarz & David Fisher

Tukia type theorems for Carnot-by-Carnot groups

Abstract:
Motivated by Mostow's proof of Mostow rigidity, Tukia proved landmark results about when a group of quasi-conformal mappings of R^n is quasiconformally conjugate to a conformal action. Gromov later pointed out that Tukia's theorem had fairly immediate consequences for the quasi-isometric rigidity of fundamental groups of hyperbolic manifolds.
In these talks we will discuss recent generalizations of Tukia's theorem to a broader class of spaces, namely what we call Carnot-by-Carnot groups. The motivation for studying this class of spaces (and even broader ones) also comes from the study of quasi-isometric rigidity for certain groups and spaces. This work is joint with Xie.

Pierre Pansu

Quasiconformal classification of contact 3-dimensional Lie groups

Abstract:
By contact Lie group, we mean the data of a Lie group and a left invariant contact structure. Any two choices of left invariant Euclidean metric on the contact structure produce quasiconformally equivalent subRiemannian metrics. Therefore the quasiconformal classification problem for contact Lie groups is well posed. We discuss the 3-dimensional case.

Georg Gruetzner

Hilbert spaces attached to Möbius spaces

Abstract:
In 2001, Y. Shalom asked if every hyperbolic group admits a uniformly bounded representation with a proper cocycle. In this talk we construct Sobolev spaces H(d) associated with a Möbius structure M. We show that M has a uniform Ahlfors-David constant, and use this observation to show that the norms on H(d) for different metrics d in the Möbius structure are comparable for a large class of functions. This is a partial result of a program to construct and study uniformly bounded representations for all hyperbolic groups.

Robert Young

Quantitative rectifiability and surfaces in Heisenberg groups

Abstract:
Intrinsic Lipschitz graphs play an important role in geometric measure theory in the Heisenberg group and other Carnot groups. Recent work has demonstrated how the shape of intrinsic Lipschitz graphs in the $2n+1$-dimensional Heisenberg group H_n depends on n. In this talk, we will describe some techniques for visualizing, constructing, and analyzing intrinsic Lipschitz graphs, explain how the quantitative rectifiability of graphs in H_n depends on n, and see how this affects geometry and analysis in H_n. Parts of this talk are joint with Naor and Chousionis-Li.

Thursday June 22, 2023

Yehuda Shalom

Conjugation invariant norms on groups

Abstract:
A conjugation invariant norm on a group induces a bi-invariant metric on it.
Two natural, yet quite extremal sources of such metrics are the discrete one,
on any (discrete) group, and a compact one, coming from a (embedding into a) compact metric group.
In the talk we shall discuss some rigidity phenomena related to these norms. In particular, we shall be interested in examples where every other norm on a discrete/compact group arises from the opposite source.
Our basic examples come from Lie and arithmetic-like groups, and we shall see how they are intimately related to deep results such as Margulis' normal subgroup theorem and Nikolov-Segal work on finitely generated profinite groups.
Based on joint work with Leonid Polterovich and Zvi Shem-Tov

Tom Ferragut

Geometry and rigidity of quasi-isometries of horospherical products.

Abstract:
Horospherical products of two Gromov hyperbolic spaces were introduced to unify the construction of metric spaces such as Diestel-Leader graphs, the Sol geometry or treebolic spaces. In this minitalk we will look at their geometry through a description of their geodesics and distances. Then we will get interested in a geometric rigidity property of their quasi-isometries.

Seung-Yeon Ryoo

Embedding balls in groups of polynomial growth into Euclidean space

Abstract:
Let Γ be a group of polynomial growth that is not virtually abelian. We show that the bilipschitz distortion of the ball of radius n in Γ into a Euclidean space of sufficiently large dimension is $\sqrt{\log n}$, up to universal constants.

Nicola Cavallucci

Convergence and collapse of CAT(0)-spaces and groups

Abstract:
I will present the possible Gromov-Hausdorff limits of geodesically complete, CAT(0)-spaces admitting a discrete group of isometries of bounded codiameter and the structure of the possible limit groups. The focus will be on the collapsing case: namely when the injectivity radius of the quotient space goes to zero along the sequence. Joint work with A.Sambusetti.

Roman Sauer

Higher property T and the cohomology of arithmetic groups

Abstract:
Property T of a group G is a fixed point property of actions of G on Hilbert spaces. It was introduced by Kazhdan in the 1960's in a most influential 3-page paper.
We study and prove a higher analog of property T for arithmetic groups which is formulated in cohomological terms. Earlier related results often relied on deep analytic and number-theoretic results. Our novel method is based on results of geometric group theory such as isoperimetric inequalities in groups. Joint work with Uri Bader.

Oussama Bensaid

Coarse separation by subsets of sub-exponential growth

Abstract:
A subset S of a metric space X is said to coarsely separate X if the complement of an R-neighborhood of S contains at least two connected components with arbitrarily large balls. We are interested in the volume growth of such separating subsets. We show that symmetric spaces of non-compact type (except the real hyperbolic plane), higher rank thick Euclidean buildings and Bourdon's hyperbolic buildings do not admit a coarse separating subset of sub-exponential growth.

Xiangdong Xie

Rigidity of maps between Carnot groups

Abstract:
We will present results showing rigidity and regularity of various maps between Carnot groups. The maps include quasiconformal maps and more generally Sobolev maps. Carnot groups appear in several branches of mathematics including geometry and analysis. In particular Carnot groups appear as the boundary at infinity of some negatively curved homogeneous manifolds. Rigidity of quasiconformal maps between Carnot groups corresponds to rigidity of quasi-isometries between negatively curved homogenous manifolds. In the first part of the talk I will give an introduction to the topic, provide the motivation and survey the known results. In the second part I will explain the ideas of some of the proofs. This talk is based on joint works with various authors including Bruce Kleiner, Stefan Muller and Enrico Le Donne.

Friday June 23, 2023

Marc Burger

Actions on asymptotic cones, non-evanescence, and real algebra

Abstract:
An important property of an action by a finitely generated group L on a complete CAT(0) space X is that its action on the visual boundary of X has no global fixed point; such an action is called non-parabolic. For instance when X is a symmetric space it guarantees the existence of equivariant harmonic maps with target X.
In general, when X is not proper, a stronger property called non-evanescence is needed. Sadly, some natural constructions involving actions don't preserve the property of being non-parabolic. This is so with taking asymptotic cones: one easily constructs a sequence of non-parabolic Z-actions on the hyperbolic plane such that the associated Z action on the asymptotic cone is parabolic. In this talk, we illustrate how techniques from real algebraic geometry can be used to show that if one takes a sequence of representations belonging to a connected component of Hom(L, Isom(X)) consisting entirely of non-parabolic actions, the resulting L-action on the corresponding asymptotic cone is non-evanescent.
Joint work with A. Iozzi, A. Parreau, B. Pozzetti.

Paolo Ciatti

Bounds for the spectral projections of the Heisenberg subLaplacian.

Abstract:
The talk will concern the boundedness of the generalized spectral projections P_λ associated to the sublaplacian in the Heisenberg group thought of as operators from $L^1_t L^p_z$ to $L^\inty_t L^q_z$. We will formulate a conjecture concerning the range of exponents p,q for which the projections P_λ are bounded on the $(2n+1)$-dimensional Heisenberg group and provide some motivations. In particular we will discuss the recent proof of the bounds on the three dimensional Heisenberg group. This work was done in collaboration with Valentina Casarino.

Luca Capogna

Conformal equivalence of visual metrics in pseudoconvex domains

Abstract:
As a segue to the topics of the minicourse, we will discuss a joint paper with Enrico Le Donne (Fribourg) in which we refine estimates introduced by Balogh and Bonk, to show that the boundary extensions of isometries between smooth strongly pseudoconvex domains in $\C^n$ are conformal with respect to the sub-Riemannian metric induced by the Levi form. As a corollary we obtain an alternative proof of a result of Fefferman on smooth extensions of biholomorphic mappings between pseudoconvex domains. The proofs are inspired by Mostow's proof of his rigidity theorem and are based both on the asymptotic hyperbolic character of the Kobayashi or Bergman metrics and on the Bonk-Schramm hyperbolic fillings.

Moon Duchin

From continuous to discrete Heisenberg geometry

Abstract:
By taking some care with sub-Finsler CC geodesics, we can build tools to answer a range of questions about the discrete Heisenberg group. In this talk I'll survey some applications that make use of the discrete-to-continuous bridge.

Nate Fisher

Horofunction boundaries of homogeneous groups

Abstract:
In this talk, I will define and motivate the use of horofunction boundaries to study groups. I will discuss some examples which demonstrate interesting properties of the horofunction boundary and share some new results about the horofunction boundaries of homogeneous groups.

Assaf Naor

On the inverse problem for isometry groups of norms.

Abstract:
Every Banach space X naturally induces a group G by considering its isometric automorphisms which fix the origin (by the Mazur—Ulam theorem, any such isometry must be linear). The corresponding inverse problem takes as input a group G and asks if it is the isometry group of some equivalent norm on X. Geometrically, when X is finite dimensional this question coincides with asking if there exists an origin-symmetric convex body in R^n whose group of symmetries is G. This talk will present some of the work that has been done on this old question, and then describe recent progress (in collaboration with Breuillard, Liebeck and Rizzoli). It turns out that the answer is positive for some groups but not for others, and it is quite subtle (and still illusive) to obtain an intrinsic characterization of which groups G admit such a representation as the isometry group of a norm. We will describe a new criterion that can be used to obtain a solution of this inverse problem in some settings by using a range of tools from a variety of areas.