Recent trends in Optimal Control and Partial Differential Equations

May 8-10, 2023

Centro di Ricerca Matematica Ennio De Giorgi
Scuola Normale Superiore, Pisa

INVITED TALKS

A new perspective on the relativistic Kolmogorov-Fokker-Planck operator

Francesca Anceschi
Università Politecnica delle Marche, Ancona

In this talk, we present a new relativistic Kolmogorov-Fokker-Planck operator introduced for the first time in a joint work with S. Polidoro and A. Rebucci. This operator is hypoelliptic and translation invariant with respect to Lorentz transformations. In this setting, we prove a Harnack inequality and asymptotic lower bounds for its fundamental solution.

Long-time behaviour of 1st order Mean Field Games systems with non-monotone interactions

Martino Bardi
Università degli Studi di Padova

We consider deterministic Mean Field Games (MFG) with a cost functional continuous with respect to the distribution of the agents and bounded away from 0 for large states of the generic player. We first show that the static MFG with such a cost has an equilibrium, and we build from it a solution of the ergodic MFG system of 1st order PDEs with the same cost. The proof is based on [1]. Earlier related results use stronger assumptions on the data [2, 3].

Next we address the long-time limit of the solutions to finite horizon MFG with cost functional satisfying various additional assumptions. The parabolic system arising when the agents are affected by independent non-degenerate noise was treated by Cardaliaguet, Lasry, Lions, and Porretta. For deterministic MFG the problem was studied by Cardaliaguet on the torus [2] and by Cannarsa et al. in the whole space [3], and both papers use in a crucial way a monotonicity condition on the cost functional. This condition strengthens the classical one by Lasry and Lions that describes a preference for less crowded areas and implies uniqueness of the solution to the MFG system. Our main assumption, instead, is about the set of minima of the cost, motivated by possible applications to global optimization, as in [1]. It allows the aggregation of the agents and the existence of multiple solutions.

We prove the convergence of the distribution of the agents and of the value function to a solution of the ergodic MFG system as the horizon of the game tends to infinity.

This research is part of an ongoing project with Hicham Kouhkouh (Univ. Aachen).

[1] M. Bardi, H. Kouhkouh: An Eikonal equation with vanishing Lagrangian arising in Global Optimization, Appl. Math. Optim. 87, 49 (2023).
[2] P. Cardaliaguet: Long time average of first order mean field games and weak KAM theory, Dyn. Games Appl. 3 (2013), 473 - 488.
[3] P. Cannarsa, W. Cheng, C. Mendico, K. Wang: Long-time behavior of first-order mean field games on Euclidean space, Dyn. Games Appl. 10 (2020), 361 - 390.

Mass concentration for Ergodic Mean-Field Games with Riesz-type aggregation

Chiara Bernardini
Università degli Studi di Padova

We consider second-order ergodic Mean-Field Games systems defined in the whole space $\mathbb{R}^N$ with Riesz-type aggregating nonlocal coupling and external confining potential V. In this setting, every player of the game is attracted toward regions where the population is highly distributed, while the external potential discourages agents to be far away from the origin. Focusing on the *mass-subcritical regime* $N - \gamma' < \alpha < N$, we show concentration phenomena in the vanishing viscosity limit, namely when the diffusion becomes negligible. First, we investigate the asymptotic behavior of rescaled solutions as $\varepsilon \to 0$, obtaining in this way existence of classical solutions to potential-free MFG systems with Riesz-type coupling. Secondly, we prove concentration of mass around minima of the potential V.

A continuous dependence estimate for viscous Hamilton-Jacobi equations on networks with applications

Fabio Camilli
Sapienza Università di Roma

We study continuous dependence estimates for viscous Hamilton- Jacobi equations defined on a network Γ. Given two Hamilton-Jacobi equations, we prove an estimate on the C^2-norm of the difference between the corresponding solutions in terms of the L^∞ distance among the coefficients. We also provide two applications of the previous estimate: the first one is an existence and uniqueness result for a quasi-stationary Mean Field Games defined on the network Γ; the second one is an estimate of the rate of convergence for homogenization of viscous Hamilton-Jacobi equations defined on a periodic network, when the size of the cells vanishes and the limit problem is defined in the whole Euclidean space.

Aubry-Mather theory for sub-Riemannian control systems

Piermarco Cannarsa
Università di Roma Tor Vergata

The long-time average behavior of the value function in the classical calculus of variation is known to be connected with the existence of solutions of the so-called critical equations, that is, a stationary Hamilton-Jacobi equation which includes a sort of nonlinear eigenvalue called the critical constant (or effective Hamiltonian). In this talks, we will address a similar issues for the dynamic programming equation of an optimal control problem, namely a control problem of sub-Riemannian type, for which coercivity of the Hamiltonian is non longer true. We introduce the Aubry set for sub-Riemannian control systems and we show that any fixed point of the Lax-Oleinik semigroup is horizontally differentiable on such a set. Furthermore, we obtain a variational representation of the critical constant by using an adapted notion of closed measures, introduced by A. Fathi and A. Siconolfi (2004). Then, defining a new class of probability measures (strongly closed measures) we define the Mather set for sub-Riemannian control systems and we prove that such a set is included in the Aubry set. We conclude this talk showing how the theory developed so far can be applied to study the well-posedness of the ergodic mean field games system defined on a sub-Riemannian structure.

Mean field games with aggregating interaction potentials of Choquard type

Annalisa Cesaroni
Università degli Studi di Padova

I will discuss existence/non existence of solutions to ergodic mean field game systems in $\mathbb{R}^n$ with interactions of aggregative Riesz type, in dependance on the strength of the interaction term.

Joint work with Chiara Bernardini.

Study of stationary equilibria in a Kuramoto Mean Field Game

Marco Cirant
Università degli Studi di Padova

In a recent work, R. Carmona, Q. Cormier and M. Soner proposed a mean field game based on the classical Kuramoto model, originally motivated by systems of chemical and biological oscillators. Such MFG model exhibits several stationary equilibria, and the question of their ability to capture long time limits of dynamic equilibria is largely open. In this talk we will show that, up to translations, there are two possible stationary equilibria only - the incoherent and the synchronised one - provided that the interaction parameter is large enough. We will then comment on their stability properties. Based on a joint work with A. Cesaroni.

Semiconcavity for the minimum time problem with time delay

Elisa Continelli
Università degli Studi dell'Aquila

We analyze a minimum time problem with time delay and we investigate the regularity properties of the value function associated to the considered optimal control problem. In our setting, the minimum time function is no longer defined in a subset of $\mathbb{R}^n$, as it happens when dealing with the undelayed case, but its domain is a subset of the Banach space $C([-\tau, 0]; \mathbb{R}^n)$. As far as the undelayed minimum time problem is concerned, it is known that, under suitable assumptions, the value function is semiconcave and it is a viscosity solution of an appropriate Hamilton-Jacobi-Belmann equation. Moreover, semiconcavity plays a prominent role in the derivation of some optimality conditions. For the value function of optimal control problems involving time delays, the Hamilton-Jacobi theory has been developed by many authors. Extending classical arguments, we are able to prove that, whenever a smallness assumption on the time delay size is required, the minimum time functional is Locally Lipschitz continuous and semiconcave with a linear moodulus in a suitable subset of the reachable set.
Joint work with C. Pignotti.

On the vanishing discount approximation for compactly supported perturbations of periodic Hamiltonians

Andrea Davini
Sapienza Università di Roma

We study the asymptotic behavior of the viscosity solutions u_G^λ of the Hamilton-Jacobi (HJ) equation

$$\lambda u(x) + G(x, u') = c(G) \qquad \text{in } \mathbb{R}$$

as the positive discount factor λ tends to 0, where $G(x,p) := H(x,p) - V(x)$ is the perturbation of a Hamiltonian $H \in C(\mathbb{R}\times\mathbb{R})$, $\mathbb{Z}$–periodic in the space variable and convex and coercive in the momentum, by a compactly supported potential $V \in C_c(\mathbb{R})$. The constant $c(G)$ appearing above is defined as the infimum of values $a \in \mathbb{R}$ for which the HJ equation $G(x,u') = a$ in $\mathbb{R}$ admits bounded viscosity subsolutions. We prove that the functions u_G^λ locally uniformly converge, for $\lambda \to 0^+$, to a specific solution u_G^0 of the critical equation

$$G(x, u') = c(G) \qquad \text{in } \mathbb{R}.$$

We identify u_G^0 in terms of projected Mather measures for G and of the limit u_H^0 to the unperturbed periodic problem. This is joint work with I. Capuzzo-Dolcetta.

Regularity of unbalanced optimal transport

Roberta Ghezzi
Università di Roma Tor Vergata

Recently, optimal transport has been extended to measures of positive and finite mass (on the Euclidean space or on a Riemannian manifold) [CSPV16, CPSV18, KMV16, LMS18], giving rise to the so-called unbalanced optimal transport. On the set of positive Radon measures on a metric space, the Wasserstein- Fisher-Rao distance is a natural generalization of the classical Wasserstein distance and admits different equivalent formulations (dynamical, semi-coupling, dual, Kantorovich formulations). In a joint work with F.-X. Vialard and T. Gallouët we show how to reduce regularity of unbalanced optimal transport to regularity of a standard optimal transport problem and we prove that unbalanced optimal transport is regular on spheres by computing the Ma-Trudinger-Wang tensor.

[CPSV18] L. Chizat, G. Peyre, B. Schmitzer, and F.- X. Vialard. Unbalanced optimal transport: dynamic and Kantorovich formulations. J. Funct. Anal., 274(11):3090–3123, 2018.
[CSPV16] L. Chizat, B. Schmitzer, G. Peyré, and F.-X. Vialard. An Interpolating Distance between Optimal Transport and Fisher-Rao. Found. Comp. Math., 2016.
[KMV16] S. Kondratyev, L. Monsaingeon, and D. Vorotnikov. A new optimal trasnport distance on the space of finite Radon measures. Adv. Differential Equations, 21(11):1117–1164, 2016.
[LMS18] M. Liero, A. Mielke, and G. Savaré. Optimal Entropy-Transport problems and a new Hellinger-Kantorovich distance between positive measures. Inventiones Math., 2018.

Existence and non-existence for time-dependent mean field games with strong aggregation

Daria Ghilli
Università di Pavia

We study second order quadratic Mean Field Games where the coupling is decreasing w.r.t. the distribution. This case captures situations in which agents aim at maximizing aggregation. While existence, uniqueness and long-time stability of solutions are known to hold when the coupling is increasing, the picture is less clear in the decreasing case. Different phenomena have been observed such as non uniqueness of solutions, periodic solutions and instability in the long-time horizon.

In particular we consider local and strongly decreasing couplings of the form

$$-\sigma m^{\alpha}, \quad \alpha \geq \frac{2}{N}$$

where m is the population density and N is the dimension of the state space. When the coupling is of the previous form, σ and α are related to the aggregation force. We prove the existence of solutions under the assumption that σ is small enough. For large σ, we show that existence may fail whenever the time horizon T is large.

This is a joint work with M. Cirant (Padua).

Liouville Theorems for Hypoelliptic Partial Differential Operators on Lie Groups

Alessia Elisabetta Kogoj
Università degli Studi di Urbino

We present several Liouville-type theorems for caloric and subcaloric functions on Lie groups in $\mathbb{R}^N$. Our results apply in particular to the heat operator on Carnot groups, to linearized Kolmogorov operators and to operators of Fokker-Planck-type like the Mumford operator. An application to the uniqueness for the Cauchy problem is also shown.

First order Mean Field Games on networks

Claudio Marchi
Università degli Studi di Padova

We focus our attention on deterministic Mean Field Games with finite horizon in which the states of the players are constrained in a network (in our setting, a network is given by a finite collection of vertices connected by continuous edges) and the cost may change from edge to edge. As in the Lagrangian approach, we introduce a relaxed notion of Mean Field Games equilibria which describe the game in terms of probability measures on trajectories instead of time-dependent probability measures on the network. Our first main result is to establish the existence of such a MFG equilibrium. Afterward, to each MFG equilibrium, can be naturally associated a cost, the corresponding value function and optimal trajectories (chosen by the agents). We prove that optimal trajectories starting at time t=0 are Lipschitz continuous, locally uniformly with respect to the initial position. As a byproduct, we obtain a "Lipschitz" continuity of the MFG equilibrium: its push-forward through the evaluation-function at each time gives rise

to a Lipschitz continuous function from the time interval to the space of probability measures on the network.

The second main result is to prove that this value function is Lipschitz continuous and solves a Hamilton-Jacobi partial differential equation in the network.

This is a joint work with: Y. Achdou (Univ. of Paris), P. Mannucci (Univ. of Padova) and N. Tchou (Univ. of Rennes).

Comparison principles for general potential theories and fully nonlinear PDEs with directionality

Kevin Payne
Università degli Studi di Milano Statale

We present some recent advances in the productive and symbiotic interplay between general potential theories (subharmonic functions associated to closed subsets $\mathcal{F} \subset \mathcal{J}^2(X)$ of the 2-jets on $X \subset \mathbb{R}^n$ open) and subsolutions of degenerate elliptic PDEs of the form $F(x, u, Du, D^2u) = 0$. We will describe the *monotonicity-duality* method begin by Harvey and Lawson [Comm. Pure Appl. Math, 2009] for proving comparison principles for potential theories where $\mathcal{F}$ has *sufficient monotonicity* and *fiberegularity* (in variable coefficient settings) and which carry over to all differential operators F which are *compatible* with $\mathcal{F}$ in a precise sense. Particular attention will be given to *gradient dependent* examples with the requisite sufficient monotonicity of *proper ellipticity* and *directionality*. Examples operators we will discuss include those of *optimal transport* in which the target density is strictly increasing in some directions as well as operators which are parabolic in the sense of Krylov. Further examples, modeled on *hyperbolic polynomials* in the sense of Gårding, produce additional examples in which the comparison principles holds, but standard viscosity structural conditions fail to hold.

Opinion formation models with time dependent time delays

Cristina Pignotti
Università degli Studi dell'Aquila

We analyze Hegselmann-Krause opinion formation models with time variable time delays. We prove that, if the influence function is always positive, then there is exponential convergence to consensus without requiring any smallness assumptions on the time delay function. We are able to deal with general influence functions, without requiring the monotonicity assumption of previous related literature. Since the convergence estimates are independent of the number N of the agents, we can extend our result to the continuity type equation obtained as the mean-field limit of the particle model when N goes to infinity. A model with heterogeneous time delays, namely the communication delays are dependent on the agents' pairs, is also considered.

Joint works with Elisa Continelli and Alessandro Paolucci.

Approximation of Nash equilibria in nonsymmetric games

Davide Francesco Redaelli
Università degli Studi di Padova

I will discuss the issue of approximating closed-loop Nash equilibria starting from open-loop equilibria in a differential game with many players. I will show a model where the value function over open-loop controls gives rise to a map which is "close to" the value function over closed-loop controls. Underlying structural assumptions focus on suitably weighted but not necessarily symmetric interactions between players. This is an ongoing joint work with Marco Cirant (Padova).

Title: TBA

Antonio Siconolfi
Sapienza Università di Roma

TBA

Uniqueness results for gauge balls in the Heisenberg group

Giulio Tralli
Università degli Studi di Padova

In this talk we consider the problem of characterizing gauge balls in the Heisenberg group by prescribing their (non-constant) horizontal mean curvature. We discuss two uniqueness results: in the lowest dimensional case under an assumption on the location of the singular set, and in higher dimensions in the restricted class of horizontally umbilical hypersurfaces. We focus on the degenerate ellipticity of the underlying operators, and on the presence/absence of Hörmander-type properties. This is a joint work with C. Guidi and V. Martino.