Scientific report on the trimester in Harmonic Analysis at the Centro de Giorgi (April 13 – July 9, 2004)

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Organizing team: Fulvio Ricci (SNS), Daniele Debertol (SNS), Oliver Dragicevic (SNS).

1. The main themes: scientific background

The initial notion that has generated the whole field of harmonic analysis is that of Fourier transform. From the very beginning, it was meant as a tool to study constant coefficient differential operators in R^n, but also more general linear operators of different kinds, sharing the common property of commuting with translations (on R^n or on the torus). Natural ranges of application of the Fourier transform, besides PDEs, turned out to be complex analysis and potential theory.
Starting with the work of Calderón and Zygmund, singular integrals, maximal operators and Littlewood-Paley-Stein decompositions assumed a crucial role in all these areas. As a matter of fact, the Calderón-Zygmund theory relies on some basic geometric/measure-theoretic facts that extend its scope far beyond the range of elliptic PDEs applications, which had motivated their investigations. So, modifying the geometry of the underlying space, one can, for instance, attack problems classically posed in one complex variable (such as the boundary behaviour of holomorphic functions in Hardy spaces on the unit disk) on pseudoconvex domains in several complex variables. Today, problems concerning the boundary bar-∂–Neumann problem or the Cauchy-Szegö projection on weakly pseudosymmetric domains pose vary challenging questions.
The methods of modern harmonic analysis have become flexible enough to adapt to a large variety of non-translation-invariant situations, the translation-invariant case remaining in the background as a model, or rather as a paradigm. Often a different kind of translation-invariance is needed to model a certain problem. This has motivated the extension of notions and tools of classical Fourier analysis to Lie groups, symmetric spaces and other geometric/algebraic structures. For instance, nilpotent groups are the right models for boundaries of pseudoconvex domains in C^n and spaces with Carnot-Caratheodory metrics (needed to study, e.g., hypoelliptic operators of Hörmander type); symmetric spaces of the non-compact type are the model for Riemannian manifolds with exponentially growing volume. A wide field of present research concerns the relations between functional calculus on Laplacians or Hörmander operators and the geometry of the underlying space.
In a different direction, the oscillatory nature of the Fourier integral suggests other kinds of extensions, to operators involving oscillatory integrals with more general phases. Again, natural models for such operators arise in the translation-invariant context and the

present level of research puts together questions posed in the classical context, such as the regularizing effect of convolution by a singular measure supported on a well-curved manifold in R^n or the restriction problem of the Fourier transform, on one side, and others arising in more general ambients, such as the properties of Radon transforms, or with L^p- or Sobolev-estimates for Fourier integral operators, on the other side. In particular, the restriction problem for the Fourier transform gives, through the “Strichartz estimates”, dispersive estimates for hyperbolic operators, which can be applied also to non-linear equations.

2. The program

The following major sub-topics were announced as the main constituents of the program:

1. Fourier multipliers, maximal operators, restriction theorems
2. Oscillatory integrals and generalized Radon transforms
3. Harmonic analysis methods in linear and non-linear PDEs
4. Analysis on Lie groups and on Carnot-Caratheodory manifolds
5. Harmonic analysis methods in several complex variables
6. Time-frequency analysis

The calendar was subdivided into shorter periods, each focused on one or two of the topics above. However, this subdivision was not meant as a strict rule, in order not to dicourage the participation of those who could not attend at the prescribed time. Another reason for doing so was that sharp edges among the various themes cannot be traced and most of the main invited guests are active in more than one of them.
The first month of the trimester overlapped with the last month of the trimester in “Analysis in phase space and applications”. Hence it has been concentrated on applications to PDEs and to the interplay with microlocal analysis (topic 3). The invitation of some of the speakers for this part of the program (P. Gérard, D. Jerison, H. Smith and D. Tataru) has been coordinated with the organizers of the other trimester.
In order to match with the format of the other trimester, each of the above speakers was asked to give a rather extended series of lectures (4 to 8 hours). For the rest of the program, a larger number of speakers was planned, each lecturing from one to three hours.
The program ultimately included 113 lecture hours, with 60 invited speakers. The total number of participants was 110. Specific grants allowed 11 among graduate students and post-docs to attend the program for an extended period (one to three months). It is regrettable that a certain number of people, who had been invited either as speakers or as junior participants, were not able to participate due to bureaucratic complications in applying for visa or to specific obligations as non-permanent residents in third Countries.
In addition to attending lectures, most participants were involved in intensive research activity, both individually and in collaboration. In at least one case (D. Phong), the speaker’s lecture included results obtained during his stay at the Centro de Giorgi. Other groups of people fruitfully worked on their research projects: Seeger-Thiele-Wright,

Auscher-Coulhon, Mauceri-Meda, Nagel-Ricci-Stein-Wainger, Krantz-Peloso, Korányi-Lohoué, to mention a few. Other collaborations originated with contact established during this trimester (Koch-Ricci). We expect that a large number of research articles will eventually ackowledge the hospitality received at the Centro de Giorgi.
In addition to the main funding provided by the Centro de Giorgi, more funds were provided through the European Project HARP (Harmonic Analysis and Related Problems), In particular, HARP supported the local expenses for many of the non-italian european speakers and provided the grants offered to the european graduate students and post-docs. Other young researchers holding post-doc grants in non-italian nodes of HARP attended the trimester on funds provided by their own nodes. In their official reports to the European Union, all of them have acknowledged their participation in the trimester as a very stimulating period, providing them a unique occasion to discuss their research problems with many of the top experts in the field.
Further funds were made available by the National Project "Analisi Armonica" (PRIN 2002), and by Università di Roma "Tor Vergata".

3. Short abstracts of the lectures

a. Developments of Calderón-Zygmund theory and geometric measure theory

Pascal Auscher (Université de Paris-Sud),
Beyond Calderón-Zygmund theory
In these talks, I will explain some new developments in the L^p theory for “singular non-integral operators” by myself and others. This goes beyond the classical work of pionners, Calderòn-Zygmund, Hörmander, Fefferman-Stein, Coifman, Weiss and others. The objective is to obtain L^p bounds in absence of bounds for kernels. They are replaced by decay measured in some $B(L^p, L^q)$ norm. The L^p bounds are obtained for arbitrary ranges of p. I also discuss the extension to weigthed norm inequalities. In the case of $div(A\nabla)$ elliptic operators, this technology gives optimal results for H^∞ functional calculi, boundedness of some Littlewood-Paley-Stein type functional, boundedness of the Riesz transforms. This also applies to some questions related to Riemannian manifolds.

Guy David (Université de Paris-Sud)
Quasiminimal sets for Hausdorff measure
I explain with two simple examples why the notion of quasiminimal sets introduced by Almgren can be useful when one tries to minimize a functional with a surface term, under a topological constraint.
Analytic capacity, Menger curvature, and rectifiability
In the first lecture I describe the connections between analytic capacity, singular integral operators, and (a little) Menger curvature. Recall that X. Tolsa recently characterized vanishing analytic capacity in geometrical terms.
In the second lecture I insist more on relations between Menger curvature, the Cauchy kernel, and rectifiability.

Steve Hofmann (University of Missouri,Columbia)

Carleson measures and elliptic operators

We discuss a general “extrapolation principle” for Carleson measures. Roughly, the idea is that if one wants to prove some scale invariant estimate on cubes (e.g., a reverse holder estimate for a weight, or a Carleson measure estimate), it is enough to show that this estimate is controlled by some Carleson measure μ, in the sense that the desired bound holds in regions where μ is small in some suitable sense. The technique was introduced by John Lewis in order to solve the Dirichlet problem for the heat equation in domains with a time-varying boundary (although we note that there are related ideas present in the earlier work of Carleson, and of David and Semmes), and has found further application to both parabolic and elliptic equations, and in particular to the analytic perturbation theory for Kato's square root operators. In these lectures, we present a recent refinement of this circle of ideas, which is joint work with J. Martell, in which we formalize a general extrapolation principle for proving reverse holder estimates. We then show how this general principle can be used, for example, to reprove the elliptic perturbation results of R. Fefferman, Kenig and Pipher.

Elena Prestini (Universita di Roma Tor Vergata)

Singular integrals with variable coefficients on product spaces

Recent progress in the theory of singular integrals on product spaces will be presented, motivated by open problems of almost everywhere convergence of double Fourier series.

Xavier Tolsa (Universitat Autónoma de Barcelona)

Analytic capacity, the Cauchy transform, and curvature of measures

In the first lecture I talk about the characterization of analytic capacity in terms of Menger curvature and its comparability with γ_+ (which in particular implies that analytic capacity is semiadditive. In the second lecture I will talk about two facts: The first one is that analytic capacity is “invariant” under bilipschitz mappings. The second one is that if the Cauchy tranform is bounded on $L^2(\mu)$, then any sufficiently smooth Calderon-Zygmund operator with odd kernel is also bounded in $L^2(\mu)$. The proof of both results relies on a corona type decomposition for non doubling measures.

Alexander Volberg (Michigan State University)

Bad measures, bad sets, but good singular operators.

Two weight Hilbert transform, noncommutative Bowen-Ruelle theorem, pressure of polynomials, and Jacobi matrices of measures of maximal entropy

A theory of nonhomogeneous Calderón-Zygmund (CZ) operators is the topic of the first and the second lectures and partially also of the third one. In the first lecture we show that the main cornerstone of the theory of CZ operators is not a cornerstone at all. Namely, one can completely get rid of homogeneity of the underlying measure. The striking application of this theory is the solution of the series of problems of Painlevé, Ahlfors and Vitushkin on the borderline of Harmonic Analysis and Geometric Measure Theory.

In the second lecture we show how the ideas from nonhomogeneous CZ theory interplay with Tolsa’s ideas of capacity theory with Calderón-Zygmund kernels to give Tolsa’s solution of the famous Vitushkin conjecture of semiadditivity of analytic capacity. We

also show what changes should be made if we want to increase the dimension and to prove the semiadditivity of Lipschitz harmonic capacity in R^n, n > 2, where the wonderful tool of Menger-Melnikov's curvature is "cruelly missing" (the expression of Guy David). We will also show why nonhomogeneous CZ theory should imply that the set of finite length and positive analytic capacity must have a non-trivial intersection with Lipschitz curve (we sketch the proof different from the original Guy David's proof).
In the third lecture we will try to show how the ideas of nonhomogeneous CZ theory are related to an old problem in the area of inverse spectral problems. The problem here belongs to Jean Bellissard and can be formulated as follows: let f be any hyperbolic polynomial with real Julia set J(f). Let us build a Jacobi matrix by harmonic measure of J(f). Then one needs to prove that this matrix is always almost periodic.
This gives a rich class of almost periodic Jacobi matrices with Cantor spectrum and singular continuous spectral measure. It has been known that for the second degree polynomials z^2-c with sufficiently large c the almost periodicity holds. For some special polynomials of degree bigger than 2, this also was known (these were some modified Tchebysheff polynomials).
Here nonhomogeneous CZ theory comes unexpectedly in the form of two weight Hilbert transform, which appears very naturally in the problem of Bellisard mentioned above.

b. Convergence of Fourier expansions and decay estimates for Fourier transforms

David Békollé (Université de Yaounde, Cameroun)
Asymptotic expansions of a new special function arising in statistical physics
We consider the new special function discovered in statistical physics by M.G. Kwato, Njock et al and called Good's functions. We discuss their asymptotic expansions.

Anthony Carbery (University of Edinburgh)
Remarks and questions around the localisation problem for n-dimensional Fourier integrals
We surveyed the known results in this area, in particular how the problem relates to L^2 weighted inequalities for the extension operator for the Fourier transform, and paying attention to the notion of Sets of Divergence for the Localisation Problem (SDLP's) and the related notion of tube-nullity. Many open questions were discussed. Most of the work was joint work with F.Soria and A. Vargas.

Leonardo Colzani (Università di Milano Bicocca)
Convergence of Fourier expansions
We study the pointwise convergence of Fourier expansions under appropriate decay conditions on the convolution kernels and differentiability conditions on the functions expanded. We also estimate the Haudorff dimension of the set where divergence may occur. In particular, when the convolution kernel is the Fourier transform of a bounded set in the plane we recover a two dimensional analog of the Dirichlet theorem on the convergence of Fourier series of functions with bounded variation. Beside Fourier integrals on Euclidean spaces, we also consider expansions in eigenfunctions of elliptic

operators on manifolds and we present examples of different behaviors between Fourier integrals, Fourier series and spherical harmonic expansions.

Alexander Iosevich (University of Missouri,Columbia)
Average decay of the Fourier transform and applications
We shall discuss some Fourier analytic inequalities related to the average decay of the Fourier transform of compactly supported measures, and their applications to problems in geometric combinatorics.

Christopher Meaney (Macquarie University)
Divergence of Jacobi and Laguerre expansions
We show that for δ below certain critical indices there are functions whose Jacobi or Laguerre expansions have almost everywhere divergent Cesaro and Riesz means of order δ.

Giancarlo Travaglini (Università di Milano Bicocca)
Average decay of the Fourier transform and irregularities of distribution
The term "irregularities of distribution" concerns the irregularity necessarily present in any way of distributing N points in the unit d-dimensional cube. Several results in this theory depend on estimates for the decay of certain Fourier transforms. We will describe some of these applications and related geometrical problems.

c. Functional calculus and spectral multipliers

Archil Gulisashvili (Ohio University)
Free propagators and Feynman-Kac propagators
Propagators or evolution families are two-parameter relatives of semigroups. They satisfy the flow condition (forward propagators) or the backward flow condition (backward propagators). Important examples of propagators are the so-called free propagators, which are integral operators generated by backward transition probability functions (forward free propagators), or by transition probability functions (backward free propagators). More examples of propagators are celebrated Feynman-Kac propagators, which are perturbations of free propagators by time-dependent functions or measures. Feynman-Kac propagators have important applications in Mathematical Physics, Probability Theory, and Partial Differential Equations. In the lecture, we will introduce classes of time-dependent measures, generalizing the Kato class on R^n, and explain the elements of the theory of Feynman-Kac propagators associated with such measures. Most of the lecture will be devoted to the inheritance problem for Feynman-Kac propagators, more precisely, we will explain in what form the properties of free propagators are inherited by their Feynman-Kac perturbations. Among the properties discussed in the lecture are the L^p-boundedness, the (L^p-L^q)-smoothing property, and the boundedness in various spaces of continuous functions (the Feller property, the Feller-Dynkin property, and the BUC-property).

Giancarlo Mauceri (Universita di Genova)

Holomorphy of spectral multipliers of the Ornstein-Uhlenbeck operator

Consider a nonnegative self-adjoint operator A on $L^2(M,\mu)$, where (M,μ) is a measure space. The set $M_p(A)$ of L^p -multipliers of A forms a Banach algebra. Necessary and sufficients conditions for membership in $M_p(A)$ have useful applications to partial differential equations, spectral theory, potential theory. In the last thirty-odd years this problem has been investigated for several operators: Laplace-Beltrami operators on Riemannian manifolds, sums of squares of vector fields, Schrödinger operators, pseudodifferential operators. I shall present some sufficient and necessary conditions for the Ornstein-Uhlenbeck operator, a "natural" Laplacian on the Euclidean space with Gauss measure, discussing in particular the necessity of the holomorphy of the multiplier in a sector.

Alan McIntosh (Australian National University)

Quadratic estimates and functional calculi of perturbed Dirac operators

This is joint work with Andreas Axelsson and Stephen Keith. We prove quadratic estimates for complex perturbations of Dirac-type operators, and thereby show that such operators have a bounded functional calculus. As an application we show that spectral projections of the Hodge-Dirac operator on compact manifolds depend analytically on L^∞ changes in the metric. We also recover a unified proof of many results in the Calderón program, including the Kato square root problem and the boundedness of the Cauchy operator on Lipschitz curves and surfaces.

Andreas Seeger (University of Wisconsin, Madison)

Problems on maximal functions and singular integrals

Given a Hörmander-Mihlin multiplier m, when does the associated maximal function define a bounded operator on L^p? We present recent joint work with M. Christ, P. Honzik, L. Grafakos on this problem.

Christoph Thiele (UCLA)

Basic analytic questions in Nonlinear Fourier analysis

Scattering transforms are often referred to as nonlinear Fourier transforms. They come in many facets, but we shall concentrate on a very simple model: consider a potential F on the real line and the corresponding Dirac operator (F-D)(F+D). Then the transmission and reflection coefficient of this operator are the nonlinear Fourier transform of F. We are interested in nonlinear analogues of regularity estimates for the classical Fourier transform such as Plancherel identity, Hausdorff young inequality, Carleson's theorem. For example, there is a nonlinear Fourier transform for functions in the space $L^2(R)$. In contrast to the classical situation, the nonlinear Fourier transform is not injective on $L^2(R)$, and one of the questions we have is to understand the fibers of this map. The work presented is joint with T. Tao.

Silei Wang (Zhejiang University)

Estimates of Wente's equation in Lorentz space

A generalized Wente's equation is concerned and the new estimates of the solution in the Lorentz spaces are obtained.

d. Oscillatory integrals, restriction theorems, and generalized Radon transforms

Michael Christ (University of California, Berkeley)
On multilinear oscillatory integrals
L^p bounds for generalized Radon transforms
New results on these two subjects are presented.

Allan Greenleaf (University of Rochester)
Estimates for overdetermined Radon transforms
I will discuss Lebesgue space estimates for averaging operators associated with variable families of curves or surfaces. In the plane, there are several different types of results, each extending a theorem of Ricci and Travaglini concerning all Euclidian motions of a convex curve in a different way. Some extensions to higher dimensions will also be considered. This is joint work with L. Brandolini and G. Travaglini.

Philip Gressman (Princeton University)
Generalized Radon transforms, optimal L^p improving properties
In this talk I will present recent work on obtaining boundary L^p-L^q estimates for Radon-like operators in various special cases. The methods are baised on work by M. Christ and are extended to deal with degenerate families of translation-invariant curves as well as non-degenerate families of variable curves in low dimensions (n=3,4). In the case of variable curves, the connections to left- and right-torsion are discussed, as well as new types of torsion which arise as the dimension increases. In particular, it will be demonstrated that the translation-invariant family of curves in 4 dimensions given by (t,t^2,t^3,t^4) is, possibly contrary to expectations, degenerate.

Michael Lacey (Georgia Institue of Technology)
Hilbert transform on vector fields
For a vector field v from the plane to the unit circle we consider the Hilbert transform H_v in direction v (with integral restricted from –1 to 1). If v has 1+ε derivatives, then H_v is a bounded operator on L^2. Essential elements of the proof are the proof of Carleson's Theorem of myself and Christoph Thiele, adapted to the current setting of the plane. A novel ingredient is a maximal function which is adapted to choice of vector field. This maximal function admits a favorable bound assuming only that the vector field is Lipschitz. This is joint work with Xiaochun Li.

Gerd Mockenhaupt (Georgia Institute of Technology)
On restriction of Fourier transforms
Hardy and Littlewood observed that L^p-spaces on the torus have the majorant property if p is a positive even integer. For p > 2 not an even integer it is known that the majorant property fails to hold. We will discuss a linearized variant of the majorant problem which relates it to *local* restriction problems for Fourier series with frequency set in [0, N].

Camil Muscalu (UCLA)
Paraproducts on the bi-disc
We discuss a generalization of the classical multilinear Coifman-Meyer theorem to the bi-parameter setting of the bi-disc. This is recent joint work with Jill Pipher, Terry Tao and Christoph Thiele.

Andreas Seeger (University of Wisconsin, Madison)
Averages over curves and related maximal functions.
We discuss sharp L^p Sobolev regularity results for averaging over curves in three dimensions, as well as results maximal functions generated by dilates of such curves.
This is joint work with Malabika Pramanik.

James Wright (University of Edinburgh)
Singular Radon transforms near L^1
In joint work with A. Seeger and T. Tao, we consider translation-invariant singular Radon transforms supported along smooth hypersurfaces whose Gaussian curvature does not vanish to infinite order. We show that these operators map LloglogL into weak L^1.

e. Harmonic analysis in discrete settings

Anthony Carbery (University of Edinburgh)
Averages in finite fields
In joint work with Jim Wright we examined analogues in the case of vector spaces over finite fields of the L^p - L^q improving and “spherical maximal” problems for averages in R^n. We gave necessary conditions on the exponents for these phenomena to hold, and, in the case of improving, demonstrated that for each k there is a class of surfaces of dimension k in F^n for which the optimal result holds. This was obtained by using A.Weil's estimates on exponential sums that were developed in his solution of the Riemann hypothesis for finite fields.

Alexandru Ionescu (University of Wisconsin, Madison)
L^p boundedness of discrete singular Radon transforms
I will discuss some recent joint work with S. Wainger concerning continuity of discrete singular Radon transforms and discrete singular integrals.

Elias M. Stein (Princeton University)
Radon-like tranforms, the Heisenberg group, and discrete analogues
The Heisenberg group gives a natural realization of the Radon transform on odd-dimensional Euclidean spaces. Using this, one is lead to consider the realated Radon-like transforms-the singular Radon transform and the maximal Radon transform. In this connection we consider their discrete analogues and describe L^2-boundedness results obtained in collaboration with A.Magyar ans S.Wainger.

Stephen Wainger (University of Wisconsin, Madison)
Some discrete problems in Harmonic Analysis

We discuss certain problems of harmonic analysis that employ methods of analytic number theory. These problems concern l^p estimates for operators defined on functions on Z^d-points in R^d with integer coordinates or discrete subgroups of homogeneous groups.
This subject was originated simultaneously by Archipov and Oskolkov and Bourgain. We will briefly discuss their work, and we will mention some recent joint results with Ionescu, Magyar and Stein.

f. Harmonic analysis methods in linear and non-linear PDEs

f1. Schrödinger operators

Patrick Gérard (Université de Paris-Sud)
The Cauchy problem for the nonlinear Schrödinger equation on compact manifolds
These lectures will be devoted to some phase space methods used in the study of the Cauchy problem for the nonlinear Schr dinger equation on Riemannian compact manifolds of dimension 2 or 3. In the first part the link between WKB method, dispersion effects and Strichartz estimates will be discussed, with applications to global existence for NLS on general compact manifolds. In the second part these results will be improved on specific manifolds such as spheres; the main tools in this case will be multilinear estimates of eigenfunctions, combined with analysis in conormal Bourgain spaces.

David Jerison (MIT)
Absence of positive eigenvalues for Schrödinger operators with rough potentials
In these lectures, I discuss joint work with A. D. Ionescu appearing in GAFA 2003. There we prove that certain Schrödinger operators with short-range, but rough potentials do not have embedded eigenvalues. For example, consider the eigenvalue equation $\Delta u+Vu = -\lambda u$ in Euclidean space. If $\lambda>0$, V belongs to $L^{n/2}(R^n)$, u belongs to $L^2(R^n)$ (the key decay at infinity that gets the uniqueness process started), and ∇u belongs to L^2 on compact subsets, then u is identically zero. In other words, there are no embedded eigenvalues λ in interior of the continuous spectrum of the operator $\Delta+V$.

Gigliola Staffilani (Stanford University)
Global well-posedness and scattering in the energy space for critical nonlinear Schrödinger equation in 3D
In this talk I will present the main steps of the proof of global well-posedness, scattering and global L^{10} spacetime bounds for energy class solutions to the quintic defocusing Schrödinger equation in 3D. This proof was recently obtained in collaboration with J. Colliander, M. Keel, H. Takaoka and T. Tao and improves upon the results of Bourgain and Grillakis, which handled the radial case. The method is similar in spirit to the induction-on-energy stategy of Bourgain, but we perform the induction analysis in both frequency space and physical space simultaneously, and replace the Morawetz inequality by an interaction variant. The principal advantage of the interaction Morawetz estimate is that it is not localized to the spacial origin and so is better able to handle non-

radial solutions. In particular, this interaction estimate together with an almost-conservation argument controlling the movement of the L^2 mass in frequncy space, rules out the possibility of energy concentration.

f2. Wave equations

Hart Smith (University of Washington)

Wave packet methods and boundary value problems

Our lectures concerned the L^p norm of eigenfunctions for metrics of limited differentiability. The results also apply to functions with spectrum contained in a unit range of frequencies. The question of interest is to bound the L^p norm in terms of a power of the frequency, given that the L^2 norm is bounded by 1. Sogge established the sharp exponent for compact Riemannian manifolds with smooth metrics, and our first result discussed was that the same exponent is valid for metrics which are only twice continuously differentiable. We then discussed the case of metrics in Hölder classes between Lipschitz and C^2, giving both examples showing that the exponents can be strictly larger, as well as establishing the best possible exponent for a range of p near 2.

Christopher Sogge (Johns Hopkins University)

Nonlinear wave equations outside of obstacles

Existence theorems for nonlinear wave equations in Minkowski space, $R\times R^3$, are usually obtained by proving coupled L^2 (energy) and L^∞ (decay) estimates for linear equations. Typically both estimates involve the invariant vector fields introduced by Klainerman. In the case of three spatial dimensions, solutions of linear wave equations decay only like 1/t, which is not integrable. Therefore, to get global existence for equations with quadratic nonlinearities, one needs additional structure, such as the null condition of Christodoulou and Klainerman, to have global solutions for small data.

The case Dirichlet wave equations outside of compact obstacles is more complicated. Here it is very difficult to prove 1/t decay unless one makes strong assumptions, such as assuming that the obstacles are star-shaped. Also, one cannot use the invariant vector fields that generate hyperbolic rotations since they have large coefficients and do not preserve the boundary conditions. In recent joint work with Jason Metcalfe, we get around this by using local exponential decay estimates of Morawetz, Ralston & Strauss as well as ones of Ikawa to prove that there is global existence outside of a wide class of obstacles that includes nontrapping ones. We also only need to assume a null condition for interactions between same-speed waves. We also present joint work with Jason Metcalfe and Ann Stewart for solutions of nonlinear wave equations in wave guides.

Daniel Tataru (University of California, Berkeley)

Phase space analysis and nonlinear dispersive equations

The aim of the course is to introduce a phase space approach to the study of linear and nonlinear dispersive equations. We first introduce the Bargman transform and a corresponding approach to the study of pseudodifferential and Fourier integral operators. Then we consider Schrödinger like equations and obtain phase space representations for

the Fourier integral operators governing the corresponding evolution. In other words, this amounts to obtaining representations of solutions as superpositions of wave packets, i.e. special solutions which are highly localized in the phase space and move essentially along the Hamilton flow. A more delicate application of this is to study long time outgoing parametrices for Schrödinger equations with variable coefficient principal part.
A second part of the course is devoted to a related approach for the wave equation. This is similar in spirit but the geometry is different. The main application that is discussed is to the study of quasilinear wave equations.

f3. Dispersive estimates

Carlos Kenig (University of Chicago)
Anderson localization for the continuous Bernoulli model and quantitative unique continuation
In these two lectures I will discuss recent work with J.Bourgain on the proof of Anderson localization for n dimensional Bernoulli models. The connection with some problems in quantitative unique continuation theory will be explained in detail.

Herbert Koch (Universität Dortmund)
Dispersive estimates and applications
Strichartz estimates for dispersive equations and related bounds for Fourier integral operators are a crucial tool in a varity of problems like well-posedness for nonlinear dispersive equations, bounds for spectral projections, absence of postive eigenvalues and Carleman inequalities. In the talks I explain a natural variable coefficient setting for dispersive estimates, the sketch of their proof and applications to spectral projections.

Andrea Nahmod (University of Massachusetts)
The Cauchy problem for the Hyperbolic-Elliptic Ishimori system
We show an improved existence result for the hyperbolic-elliptic nonlinear dispersive system proposed by Ishimori in analogy with the 2d CCIHS chain. The proof uses gauge geometric tools and energy estimates in combination with a new method devised by C. Kenig to obtain a priori $L^q_t L^\infty_x$ estimates for classical solutions to certain dispersive equations.

Philippe Tchamitchian (Universit d’Aix-Marseille III)
Koch and Tataru solutions to Navier-Stokes equations in the space, asymptotic behaviour and stability property
In the absence, at the present time, of any satisfactory result on the existence of global, unique and regular solutions to homogeneous incompressible Navier-Stokes equations in the space, for a large enough class of initial data, a simpler question naturally arises: what can be said about the topology of the set of those initial data leading to such a good solution? In particular, is it *open*? Answering affirmatively means proving a stability result, with respect to perturbations on the Cauchy data, of the kind we are interested in here. Our starting point is a recent paper by Kawanago, later extended by Gallagher, Iftimie and Planchon. In these lectures we elaborate on Gallagher, Iftimie and Planchon

paper in order to reach the main case which is out of the scope of their results, that of Koch and Tataru solutions. For this purpose, we abandon Littlewood-Paley techniques and instead use simple real variable estimates. Since Koch and Tataru construction is optimal, our result is optimal, too, implying all the stability results previously known for these equations.

Sarah Ziesler (Dominican University)
Maximal Function Estimates with Applications to a modified Kadomstev-Petviashvili Equation

g. Analysis on Lie groups and on Carnot-Carathéodory manifolds

g1. Functional calculus of sublaplacians

Andrzej Hulanicki (University of Wroclaw)
Invariant measures for Gaussian processes on nilpotent Lie groups
We consider a group S=NA, with group multiplication in S given by (x,a)(x',a')=(xϕ_a(x'),aa'), where the ϕ_a form a one-parameter group of shrinking automorphisms of the nilpotent Lie group N. By "shrinking" we mean that ϕ_a(x') tends to the identity if a tends to 0. We discuss the properties of the semigroup generated by a class of sub-Laplacians with drift on S and present a result obtained jointly with Ewa Damek.

Stefano Meda (Universita di Milano Bicocca)
Sublaplacians with drift on Lie groups
We study spectral multipliers of right-invariant sublaplacians L with drift on an amenable, connected Lie group G. The operators we consider are self-adjoint with respect to a measure $\chi\lambda_G$, whose density with respect to the left Haar measure λ_G is a nontrivial positive character χ of G. We show that if $p\neq2$, then every $L^p(\chi\lambda_G)$-spectral multiplier of L extends to a bounded holomorphic function on a parabolic region in the complex plane, which depends on p and on the drift. When G is of polynomial growth we show that this necessary condition is nearly sufficient, by proving that bounded holomorphic functions on the appropriate parabolic region which satisfy mild regularity condition on its boundary are $L^p(\chi\lambda_G)$ multiplier of L.

Detlef Müller (Universität Kiel)
Sub-Laplacians of holomorphic L^p-type on Lie group
We study the problem of determining all connected Lie groups G which have the following property (hlp): every sub-Laplacian L on G is of holomorphic L^p type for $1 \leq p < \infty$, $p \neq 2$. First we show that semi-simple non-compact Lie groups with finite center have this property, by using holomorphic families of representations in the class one principal series of G and the Kunze-Stein phenomenon. We then apply an L^p-transference principle for induced representations, essentially due to Anker, to show that every connected Lie group G whose semisimple quotient by its radical is non-compact has property (hlp). One is thus reduced to studying those groups for which the semi-simple

quotient is compact, i.e. to compact extensions of solvable Lie groups. We consider semi-direct extensions of exponential solvable Lie groups by connected compact Lie groups. It had been proved in previous joint work with Hebisch and Ludwig that every exponential solvable Lie group S, which has a non-* regular co-adjoint orbit whose restriction to the nilradical is closed, has property (hlp), and we show here that (hlp) remains valid for compact extensions of these groups.

Singular spherical maximal operators on a class of two step nilpotent Lie groups

Let H_n be the Heisenberg group and let μ_t be the normalized surface measure for the sphere of radius t in R^{2n}. Consider the maximal function defined by $Mf = \sup_{t>0} |f * \mu_t|$. We prove that for $n \geq 2$ M defines an operator bounded on $L^p(H_n)$ provided that $p > 2n/(2n - 1)$. This improves an earlier result by Nevo and Thangavelu, and the range for L^p boundedness is optimal. We also extend the result to a more general setting of surfaces and to groups satisfying a nondegeneracy condition; these include the groups of Heisenberg type.

g2. Analysis on symmetric spaces and Riemannian manifolds

Jean-Philippe Anker (Universit de Orl ans)

Sharp heat kernel bounds on noncompact symmetric spaces

The talk contains a review of the known results on pointwise estimates for heat kernels on symmetric spaces of the non-compact type, and it includes original results of Anker and Ostellari in the higher rank case.

Riesz transforms on the Iwasawa NA subgroup of SL(3,C)

We shall report on joint work in progress with Garth Gaudry and Peter Sjögren. We consider first and second order Riesz transforms on the Iwasawa NA group in G = SL(3, C). More precisely we investigate the integrability at infinity of their convolution kernels. The results are significantly different from the rank one case studied before.

Thierry Coulhon (Université de Cérgy-Pontoise)

Riesz transform on non-compact Riemannian manifolds and heat kernel regularity

These lectures are about a joint work with Pascal Auscher, Xuan Thinh Duong, and Steve Hofmann. Its aim is to give a necessary and sufficient condition for the two natural definitions of homogeneous first order L^p Sobolev spaces to coincide on a large class of Riemannian manifolds, for p in an interval (q_0,p_0), where $2<p_0<\infty$. For non-compact manifolds, and again $p_0=\infty$, a sufficient condition has been asked for by Robert Strichartz in 1983 and many partial answers have been given since. The condition we propose is in terms of regularity of the heat kernel, more precisely in terms of integral estimates of its gradient. We are able to treat manifolds with the doubling property together with natural heat kernel bounds, as well as the ones with locally bounded geometry where the bottom of the spectrum of the Laplacian is positive.

Noel Lohoué (Université de Paris-Sud)

Analysis on non-Riemannian symmetric spaces

Let G be a semisimple Lie group and H a closed subgroup such that G/H is symmetric and non-Riemannian. We sketch proofs of some recent results such as the Hardy-Littlewood theorem and the Clerc-Stein multiplier theorem. For Kazhdan groups of rank grater than 2, our results follow from the Kazhdan property, but our method gives sharp constants.

Duong Phong (Columbia University)
Monge-Ampère equations, stability, and asymptotics for energy functionals
We discuss the problem of finding Kähler-Einstein metrics on manifolds of positive first Chern class. This reduces to the solution of Monge-Ampère equations, whose existence has been conjectured by S.T. Yau to be equivalent to stability in the sense of geometric invariant theory. We discuss the relation of stability to energy functionals, and the exact evaluation of these for complete intersections.

g3. Carnot-Carathéodory geometry

Luca Capogna (University of Arkansas)
A mean curvature flow in the Heisenberg group
The gradient flow of the "Carnot-Carathéodory" perimeter in the Heisenberg group is represented by a PDE system reminescent of the classical mean curvature flow. We will discuss some recent work joint with Mario Bonk (U. of Michigan) concerning self-similar solutions and the behaviour of legendrian foliations.

Michael Cowling (University of New South Wales)
Regularity of 1-quasiconformal maps in Carnot groups and in Carnot-Carathéodory manifolds
The first of these two talks describes the geometry of Carnot groups and Carnot-Carathéodory manifolds, and establishes that 1-quasiconformal maps beween domains in Carnot groups are regular. The second discusses the analogous problem for Carnot-Carathéodory manifolds, and relates this problem to the theorem of Fefferman on biholomorphic maps of pseudoconvex domains

Bruno Franchi (Università di Bologna)
Submanifolds of the Heisenberg group
In this talk we provide a sketch of some recent joint results with R. Serapioni and F. Serra Cassano, concerning an intrinsic definition of submanifolds of the Heisenberg group, as well as characterization of the intrinsic Hausdorff measure concentrated on these submanifolds. It turns out that the notion of submanifold deeply changes passing from "low dimension" to "low codimension".

Adam Koranyi (CUNY)
Liouville-type theorems in parabolic geometry
Parabolic geometry is the intrinsic geometry of a homogeneous space G/P where G is a simple Lie group and P a parabolic subgroup. The conformal geometry of the sphere is a special instance of such a geometry. The subject of the talk is the characterization of the

action of G by local geometric properties in the spirit of the classical Liouville theorem on conformal mappings.

h. Harmonic analysis methods in several complex variables

Steven Krantz (Washington University)
Analysis on the worm domain
In joint work with Marco Peloso, we study the Bergman kernel on a version of the Diederich/Fornaess worm domain. We are able to obtain an asymptotic expansion for the kernel, and to study its mapping properties. We can recover versions of recent irregularity results of M. Christ and E. Ligocka.

Loredana Lanzani (University of Arkansas)
Cauchy integral representations on non-smooth domains in several variables
We review integral representation methods for holomorphic functions on convex or strictly pseudoconvex domains in two or more complex variables, and discuss work in progress (with D. Barrett) for certain non-smooth domains.

Alexander Nagel (University of Wisconsin, Madison)
Strong maximal functions defined by monomial inequalities
Diagonal estimates for the Bergman kernel in some non-diagonalizable domains
We study the maximal function relative to a multi-parameter family of sets defined by monomial inequalities.
Diagonal estimates for the Bergman kernel in some non-diagonalizable domain
We obtain uniform estimates for the Bergman kernel on the diagonal in some non-diagonalizable domains including the “cross of iron”.

Marco Peloso (Politecnico di Torino)
L^p-boundedness of the Bergman projection on a worm domain
This is joint work with Steven Krantz. In this seminar I will consider a non-smooth, unbounded version of the Deiderich-Fornaess worm domain, D_β. It is known that on such domains the Bergman projection does not preserve the Sobolev space W^k, when k is larger than a certain k_0 depeding on the winding of the domain. I will indicate how to compute the asymptotic expansion of the Bergman kernelon D_β. Having such an expansion, it is possible to determine the exact range of p's for which the Bergman projection is a bounded operator on the Lebesgue space $L^p(D_\beta)$.

i. Time-frequency analysis

Loukas Grafakos (University of Missouri, Columbia)
Distributional estimates for the Carleson-Hunt operator and the bilinear Hilbert transform
Time-frequency analysis is used to prove an estimate of Hunt concerning the distribution function of the Carleson-Hunt operator acting on characteristic functions of measurable sets. An analogous estimate is proved for the bilinear Hilbert transform.

Martin Reimann (Universitát Bern)
Mathematical models for the cochlea: wavelets and the affine group
The hearing process is based on time and scale invariance. Mathematical models for the cochlea (our inner ear) have to respect this invariance. In signal processing the wavelets are the tools which display the same invariance behaviour. In this talk I consider the question How can wavelets be used in the mathematical models for the cochlea?

Guido Weiss (Washington University)
A unified characterization of reproducing systems for square integrable functions on n-dimensional Euclidean space; various applications of this characterization.
The Gabor systems are obtained by the action of a countable family of modulations and translations on an appropriate square integrable fuction, similarly, wavelet systems are obtained in an analogous fashion by using translations and dilations. There are many different systems that involve the action of a countable family made up of appropriate collections of translations, modulations and dilations. We state and prove a general theorem that characterizes all these various systems. In the second lecture we show how this general result can be applied in order to obtain a wide variety of such systems that are orthonormal bases or, more generally, Parseval frames.

j. Combinatorial problems in harmonic analysis

Alexander Iosevich (University of Missouri, Columbia)
Combinatorial complexity of convex sequences
Let {b(j)} be a convex sequence of N real numbers. Let M_d denote the number of solutions of the equtions $b(j_1)+\ldots.+b(j_d)=b(j'_1)+\dots+b(j'_d)$. We will show that M_d does not grow faster than N to the power $2d-2+2^{-d+1}$ without any additional assumptions on the sequence b(j). This is joint work with M. Rudnev and V. Ten. The result was independently obtained by S. Konyagin using different methods.

Michail Kolountzakis (University of Crete)
The Fuglede Conjecture
We will describe progress on the Fuglede Conjecture (are the domains which tile space by translation the same as those whose L^2 space admits an orthogonal basis of exponentials $\exp(2\pi i\lambda x)$?) during the past 4-5 years, and recent developments including Tao's counterexample in dimension 5.

Izabella Laba (University of British Columbia)
Distance sets corresponding to non-Euclidean norms
Let X be the 2-dimensional plane equipped with a non-Euclidean norm in which the unit ball is a convex set K, and let S be a well-distributed subset of X. We address the question of how small the distance set of S in X can be, depending on properties of K. In particular, we prove that there is a well-distributed S whose distance set has bounded density if and only if K is a polygon with finitely many sides, all of which have algebraic slopes in some coordinate system. We also consider the “continuous” version of the

problem, i.e. given a planar set E of positive Hausdorff dimension s, how does the dimension of its distance set in X depend on s and on the properties of K? The results presented in this talk were obtained jointly with Alex Iosevich and with Sergei Konyagin.